Relativistic effects

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Large-scale effects

- We have neglected **large-scale effects** in the derivation of the density fluctuation in redshift-space.

- We have neglected terms in the **redshift**

$$1+z=\frac{a_O}{a_S}\left[1+\mathbf{v}_S\cdot\mathbf{n}-\Psi_S-\int_0^{r_S}dr(\dot{\Phi}+\dot{\Psi})\right]$$

- We have neglected terms in the calculation of the **Jacobian**

$$\frac{\partial r_{\rm obs}}{\partial r}=1+\frac{1}{\mathcal{H}}\partial_r\left(\mathbf{v}\cdot\mathbf{n}\right)+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\mathbf{v}\cdot\mathbf{n}$$

Hierarchy

- The perturbations can be **classified**:

$$\delta \sim \left(\frac{k}{\mathcal{H}}\right)^2 \Phi \qquad v \sim \frac{k}{\mathcal{H}}\Phi \qquad \Psi \sim \Phi$$

- Gradients: $\partial_r \to k$

$$\frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n}) \sim \left(\frac{k}{\mathcal{H}}\right)^2 \Phi$$

$$\frac{1}{\mathcal{H}}\partial_r\Phi \sim \frac{k}{\mathcal{H}}\Phi$$

- Well **inside** the **horizon**: $\frac{k}{\mathcal{H}} \gg 1$

- Density and redshift distortions dominate

$$\delta_{\rm obs} = \delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n}) \sim \left(\frac{k}{\mathcal{H}}\right)^2 \Phi$$

Near the horizon

- What happens at **very large scales**?
- Modes outside the horizon, with $\frac{k}{\mathcal{H}} < 1$ are not observable.
- Large surveys will observe modes **near horizon**, with $\frac{k}{\mathcal{H}} \gtrsim 1$

$$\Phi \lesssim \frac{k}{\mathcal{H}}\Phi \lesssim \left(\frac{k}{\mathcal{H}}\right)^2 \Phi$$

non-negligible corrections dominant

- Current surveys are not affected by the large-scale modes due to their limited sky coverage.
- Future surveys like **Euclid** will observe much larger volumes and may be sensitive to **relativistic corrections**.

Relativistic derivation

We need to calculate the observed density fluctuation without neglecting large-scale contributions → **relativistic calculation**, beyond Newtonian approximation.

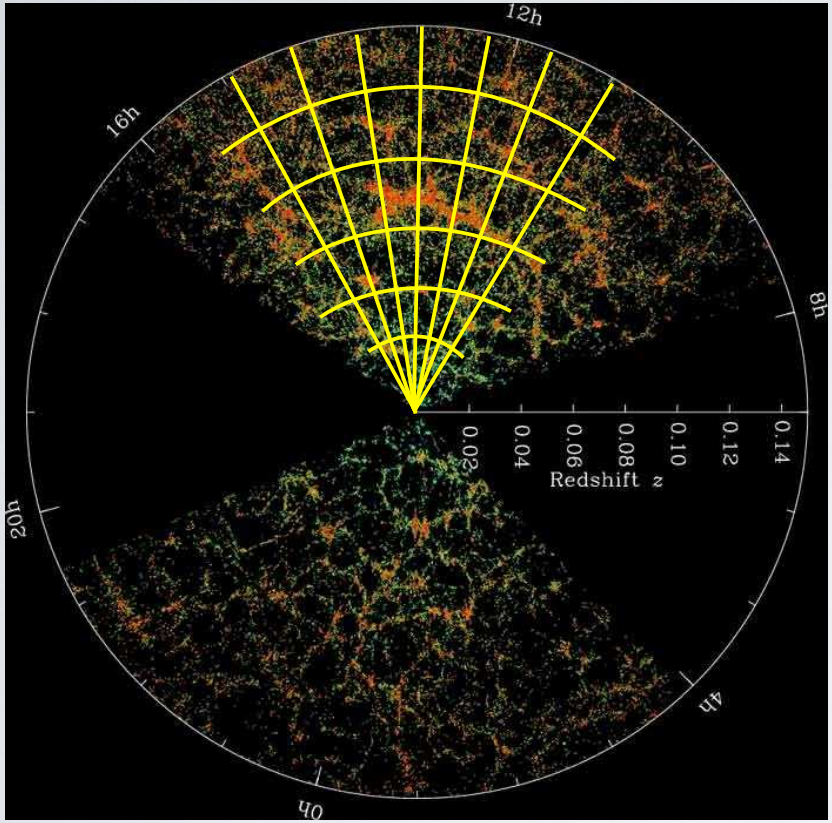
- We cannot neglect large-scale terms in Einstein's and conservation equations: $\delta'_{dm} = kv_{dm} + 3\Phi'$

- In general relativity the space-time is **four-dimensional**: a galaxy which is further away has emitted light earlier.

- Light does **not** travel on **straight lines** → lensing modifies the observed distribution of galaxies.

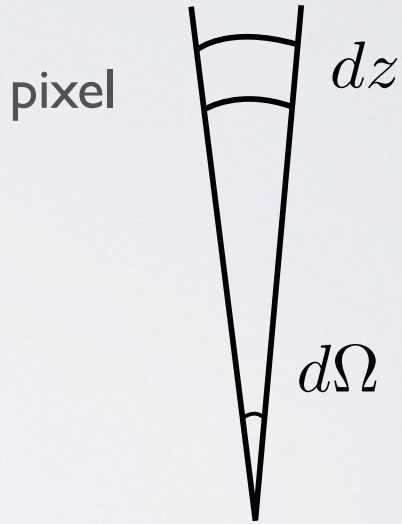
What are we observing?

We count the number of **galaxies** per **pixel**.



Observed coordinates:

- redshift z
- direction of incoming light $\mathbf{n}$ (θ, φ)



We need to understand how our **observables** (number of galaxies, solid angle and redshift bin) are affected by **fluctuations** in the density, velocity and gravitational potential.

What are we observing?

- We count $N(z, \mathbf{n})$ **galaxies** in the **volume** element $V(z, \mathbf{n})$
- We want to calculate the fluctuations in $N(z, \mathbf{n})$ with respect to the average number.
- At each redshift, we average over the directions: $\bar{N}(z)$
- The observed **over-density** is:

$$\delta_{\rm obs} = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}$$

- Relation with the matter energy density:

$$N(z, \mathbf{n}) = \rho(z, \mathbf{n}) \cdot V(z, \mathbf{n}) \quad \text{and} \quad \bar{N}(z) = \bar{\rho}(z) \cdot \bar{V}(z)$$

Derivation

$$\bar{\rho} + \delta\rho \qquad\qquad \bar{V} + \delta V$$

$$\delta_{\rm obs} = \frac{\rho(z,\mathbf{n}) \cdot V(z,\mathbf{n}) - \bar{\rho}(z) \cdot \bar{V}(z)}{\bar{\rho}(z) \cdot \bar{V}(z)}$$

We neglect non-linear terms:

$$\delta_{\rm obs} = \frac{\delta\rho(z,\mathbf{n})}{\bar{\rho}(z)} + \frac{\delta V(z,\mathbf{n})}{\bar{V}(z)}$$

the background redshift is different
from the observed redshift

$$\delta(z,\mathbf{n}) \equiv \frac{\rho(z,\mathbf{n}) - \bar{\rho}(\bar{z})}{\bar{\rho}(\bar{z})} \neq \frac{\delta\rho(z,\mathbf{n})}{\bar{\rho}(z)} = \frac{\rho(z,\mathbf{n}) - \bar{\rho}(z)}{\bar{\rho}(z)}$$

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Derivation

$z = \bar{z} + \delta z$ Taylor: $\bar{\rho}(z) = \bar{\rho}(\bar{z} + \delta z) \simeq \bar{\rho}(\bar{z}) + \partial_z \bar{\rho} \cdot \delta z$

$$\frac{\delta\rho(z, \mathbf{n})}{\bar{\rho}(z)} = \frac{\rho(z, \mathbf{n}) - \bar{\rho}(z)}{\bar{\rho}(z)} \simeq \frac{\rho(z, \mathbf{n}) - \bar{\rho}(\bar{z}) - \partial_z \bar{\rho} \cdot \delta z}{\bar{\rho}(\bar{z})}$$

$$\frac{\delta\rho(z, \mathbf{n})}{\bar{\rho}(z)} = \delta(z, \mathbf{n}) - 3\frac{\delta z}{1 + z}$$

$$\boxed{\delta_{\rm obs}(z, \mathbf{n}) = \delta(z, \mathbf{n}) + \frac{\delta V(z, \mathbf{n})}{V} - 3\frac{\delta z}{1 + z}}$$

describes what we observe in the linear regime

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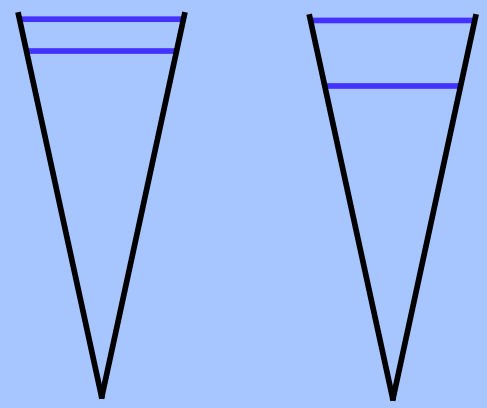
same redshift bin
different physical volume



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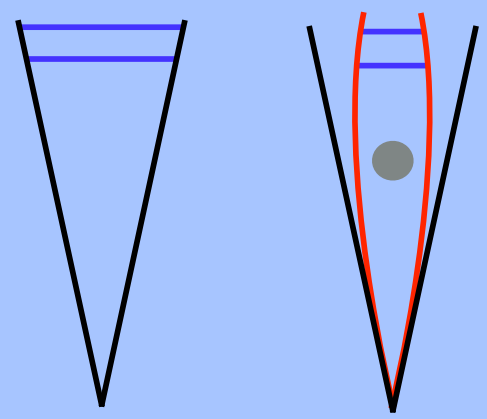
same solid angle
different physical volume



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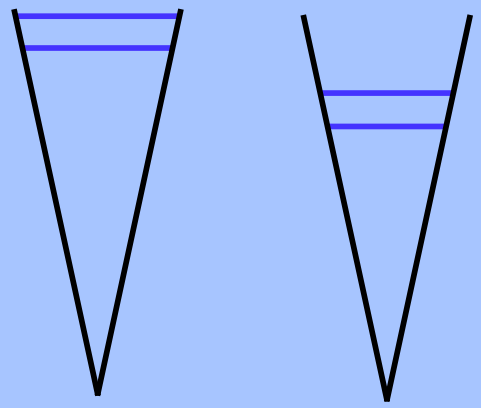
$$\frac{\delta\rho(z,\mathbf{n})}{\bar{\rho}(z)} = \delta(z,\mathbf{n}) - 3\frac{\delta z}{1+z}$$

same radial bin
different distance



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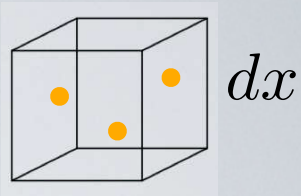
describes what we observe in the linear regime

Volume element

gives the direction of time for the galaxies inside the volume



$$dV = \sqrt{-g}\,\epsilon_{\alpha\beta\gamma\sigma}\,u^\alpha \mathrm{d}x^\beta \mathrm{d}x^\gamma \mathrm{d}x^\sigma$$

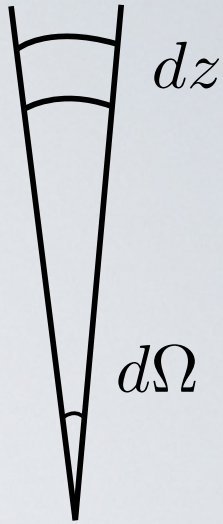
3 dimensional volume in a 4 dimensional space time

$$dV = \sqrt{-g}\,\epsilon_{\alpha\beta\gamma\sigma}\,u^\alpha \frac{\partial x^\beta}{\partial z}\frac{\partial x^\gamma}{\partial \theta_S}\frac{\partial x^\sigma}{\partial \varphi_S}\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right| \mathrm{d}z\mathrm{d}\theta_O\mathrm{d}\varphi_O$$

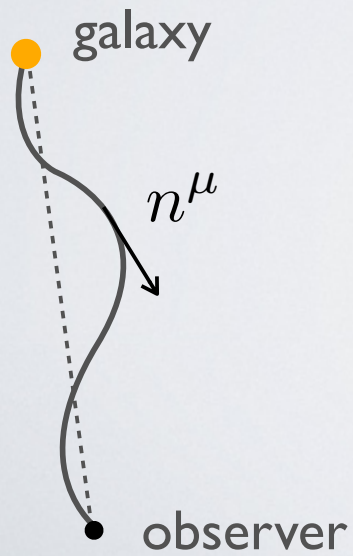
physical volume

perturbations

observed volume



$$\theta_S = \theta_O + \delta\theta \qquad \varphi_S = \varphi_O + \delta\varphi \qquad r = \bar{r} + \delta r \qquad z = \bar{z} + \delta z$$



The **null geodesic** equation tells us how directions and distances are **perturbed**.

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

Result

density

redshift space distortion

lensing

gravitational redshift

Doppler

$$\delta_{\rm obs}(z,\mathbf{n}) = b\cdot D_s - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$
$$- \int_0^r dr' \frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$
$$+ \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi \qquad \frac{\mathcal{H}}{k}D_s$$
$$\left.\begin{aligned} &+ \Psi - 2\Phi + \frac{1}{\mathcal{H}}\dot{\Phi} - 3\frac{\mathcal{H}}{k}V + \frac{2}{r}\int_0^r dr'(\Phi+\Psi) \\ &+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)\left[\Psi + \int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right] \end{aligned}\right] \rightarrow \left(\frac{\mathcal{H}}{k}\right)^2 D_s$$

potential

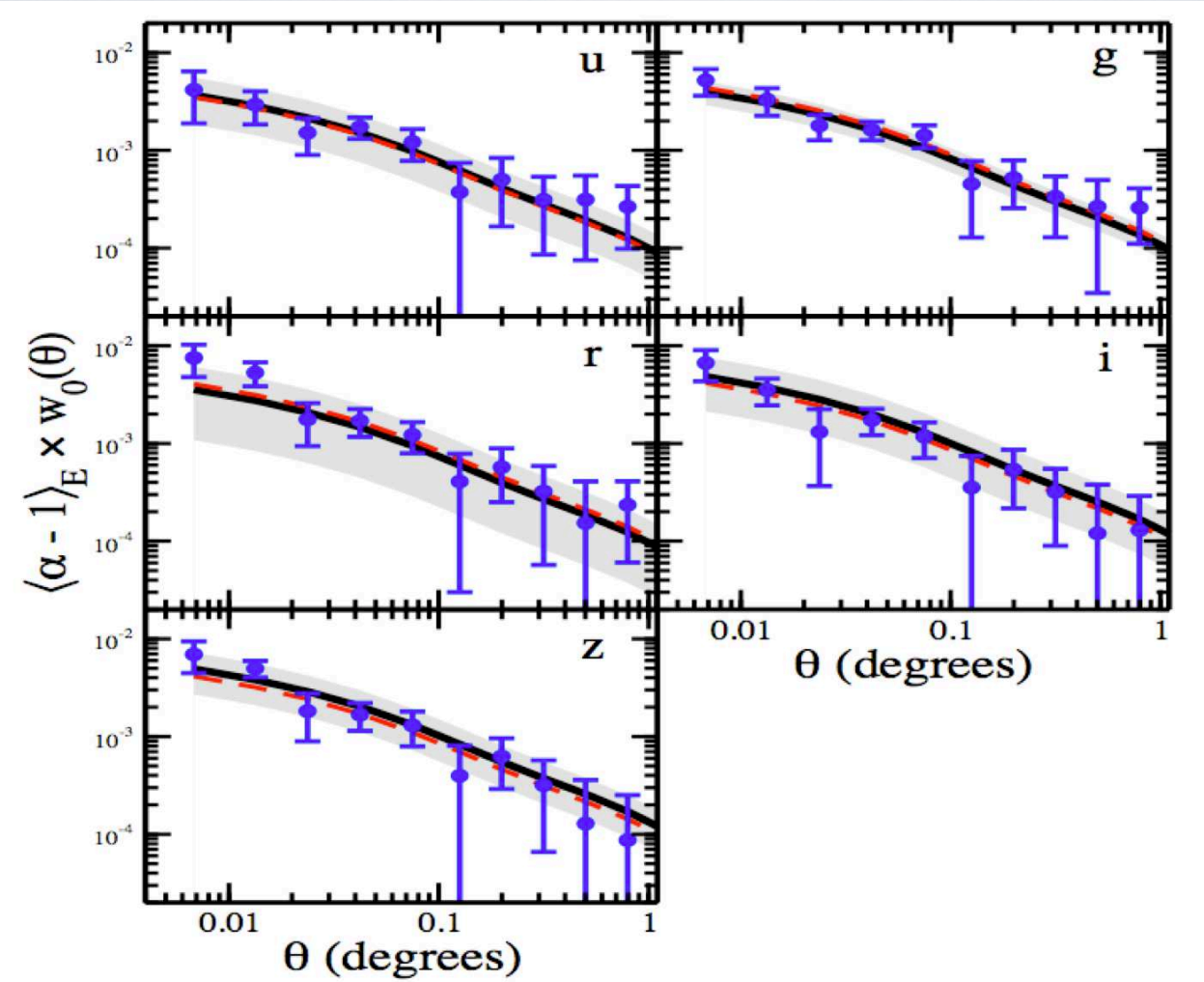
Lensing

- **Lensing** describes the change in the solid angle of observation.
- In real surveys, it combines with another similar effect: **magnification bias**.
- Galaxy surveys are **limited** in **magnitude**: we detect galaxies brighter than a given threshold.
- **Lensing** changes the magnitude: galaxies behind a cluster appear **brighter**.

$$\delta_{\rm lens} = \left(\frac{5s}{2} - 1\right) \int_0^r dr' \frac{r - r'}{rr'} \Delta_\Omega (\Phi + \Psi)$$

Observations

Correlation between background **quasars** and foreground galaxies.



Credit: Scranton et al. (2005)

Large-scale effects

$$\delta_{\rm rel} = \frac{1}{\mathcal{H}}\partial_r \Psi + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}$$

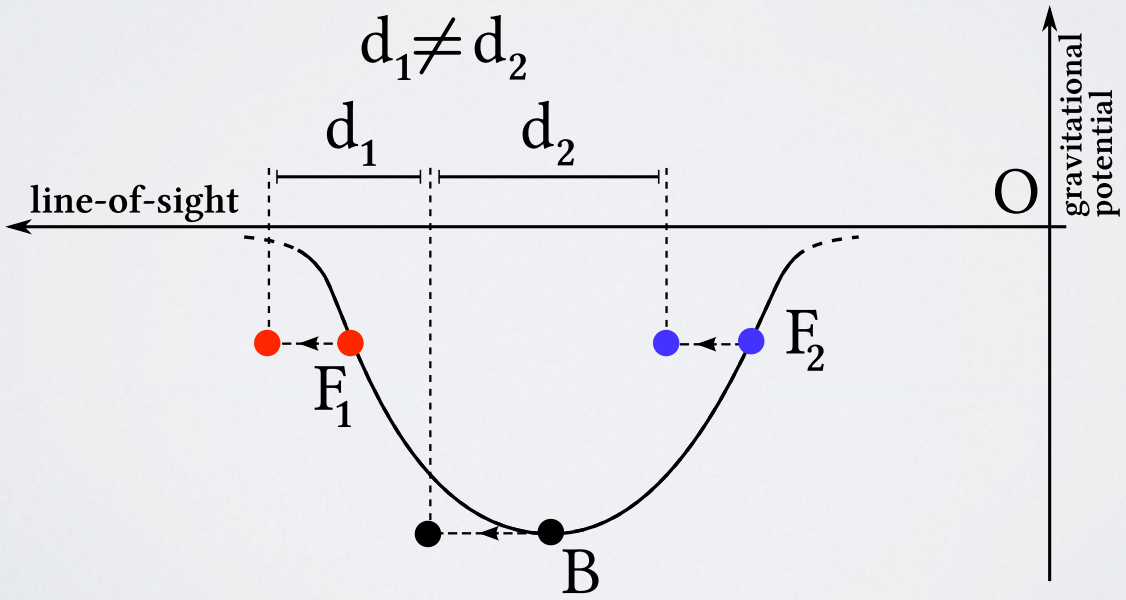
The relativistic effects break the **symmetry**: the correlation function depends on which galaxy is in **front** or **behind**.

Redshift distortion breaks the **isotropy**, but not the symmetry.

Anti-symmetry

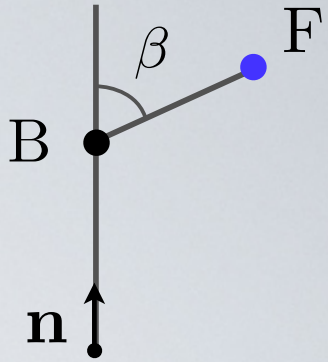
- We need **different galaxies**.
- One possibility: separate galaxies into two populations, based on their luminosity, i.e. **bright** and **faint** galaxies.
- We measure the cross-correlation between bright and faint: different when the faint is **behind** or in **front** of the bright.

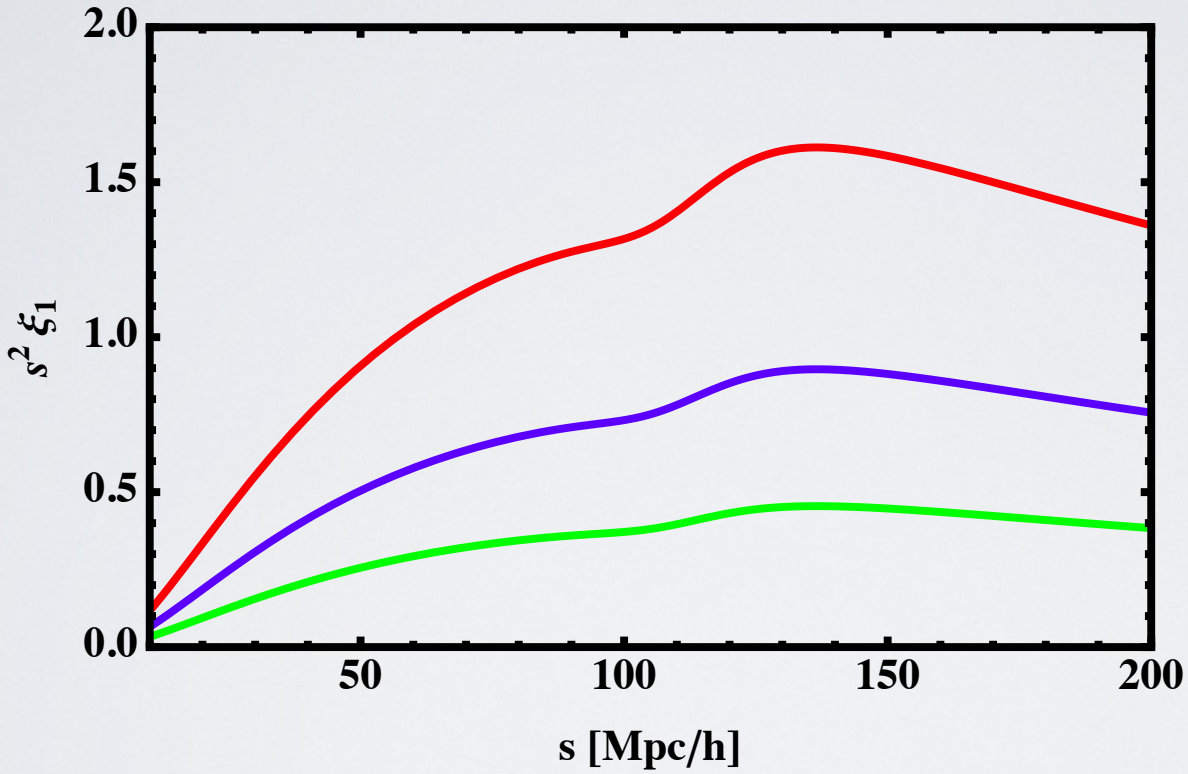
→ The correlation is **stronger** when the faint is **behind** the bright.



Dipole in the correlation function

$$\xi(r,s,\beta) = D_1^2 f \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)(b_B - b_F)\nu_1(s)\cos(\beta)$$





$z = 0.25$

$z = 0.5$

$z = 1$

Can we **measure** the dipole?