Dark energy and redshift-space distortions

Lectures 3&4

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Summary

Evolution of dark matter fluctuations from inflation until today.

- Initial conditions: $\langle \Phi_p(\mathbf{k})\Phi_p(\mathbf{k}')\rangle \sim \frac{1}{k^3}\,\delta_D(\mathbf{k}+\mathbf{k}')$

- Evolution of the gravitational **potential** governed by:

$$\Phi'' + 3(1+c_S^2)\mathcal{H}\Phi' + c_S^2 k^2\Phi = 0$$

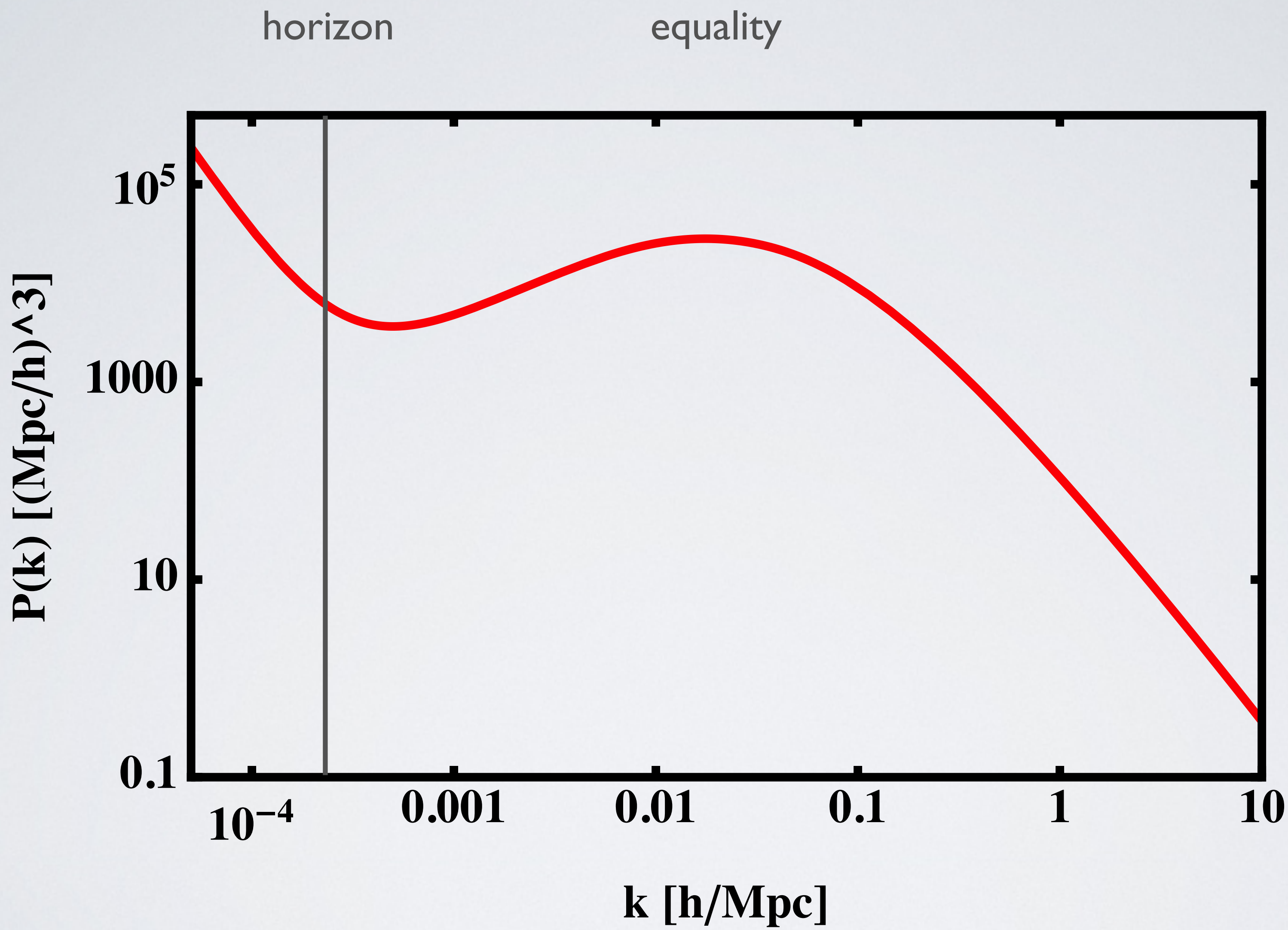
Radiation: oscillations and decay.
Matter: constant (gravity balanced by expansion)

- Evolution of the **dark matter fluctuations** governed by:

$$\delta''_{dm} + \mathcal{H}\delta'_{dm} = -k^2\Phi + 3\mathcal{H}\Phi' + 3\Phi''$$

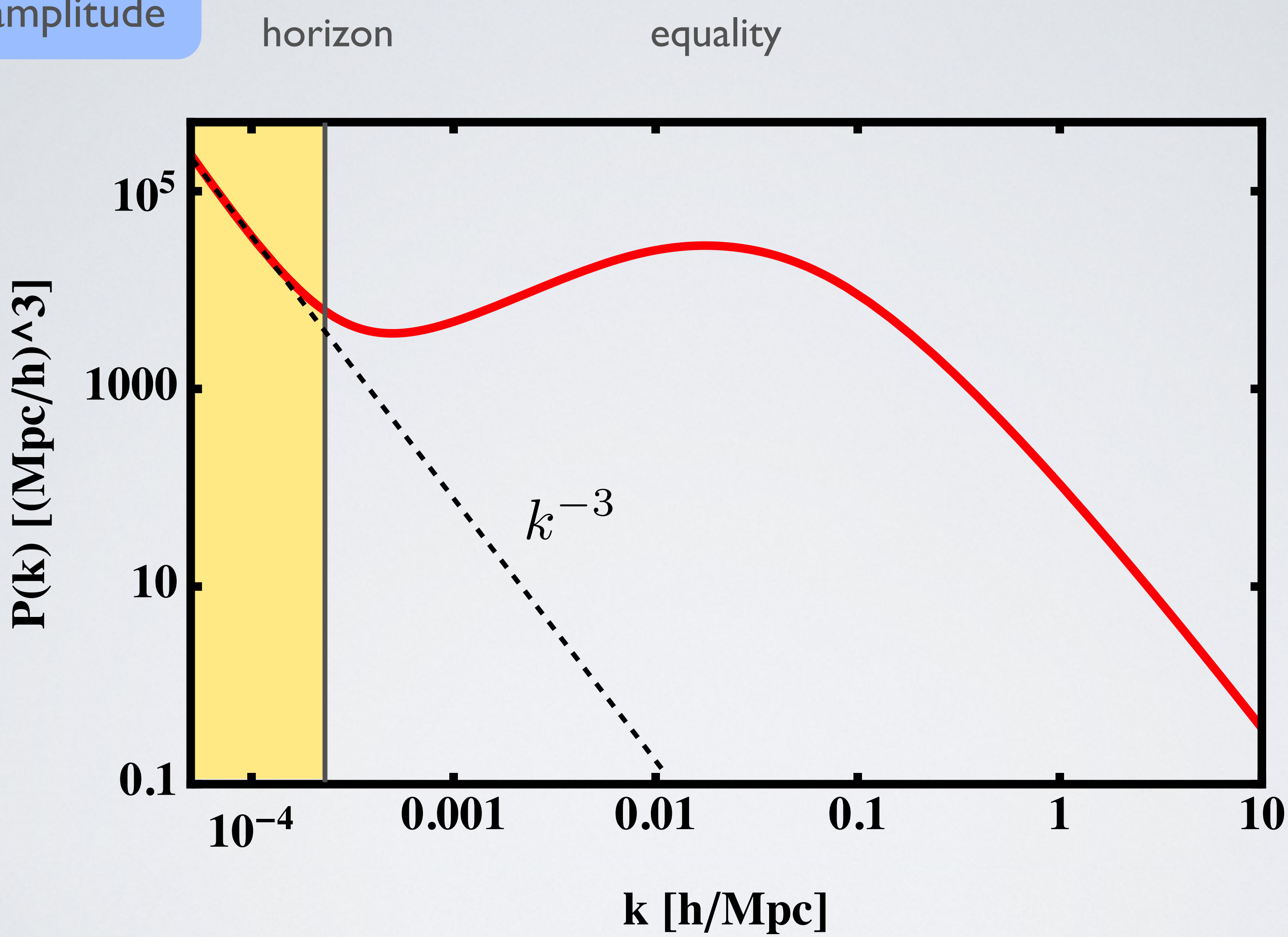
Radiation: logarithmic growth.
Matter: linear growth

Power spectrum



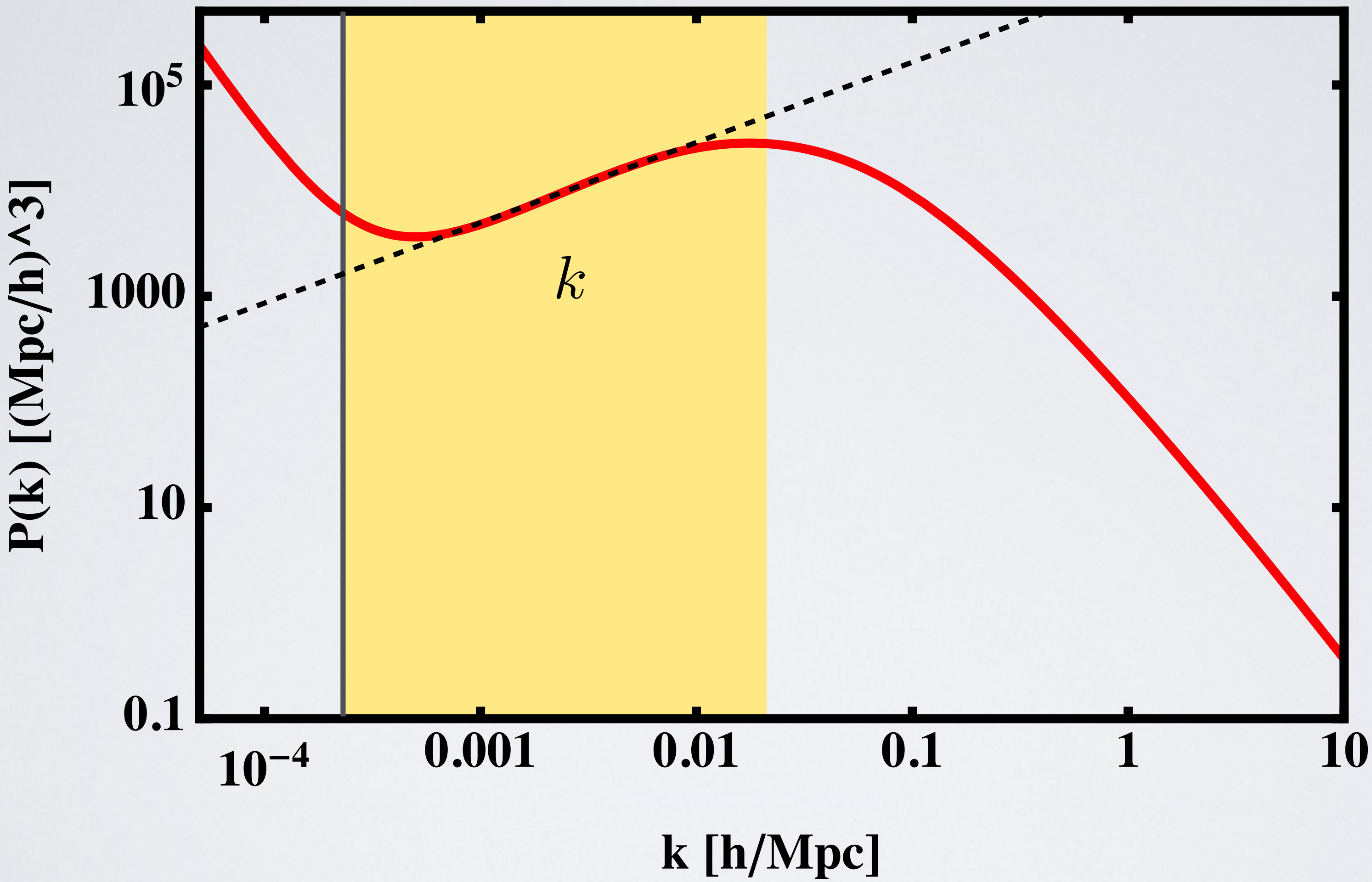
Power spectrum

Super-horizon scales: governed by the primordial amplitude



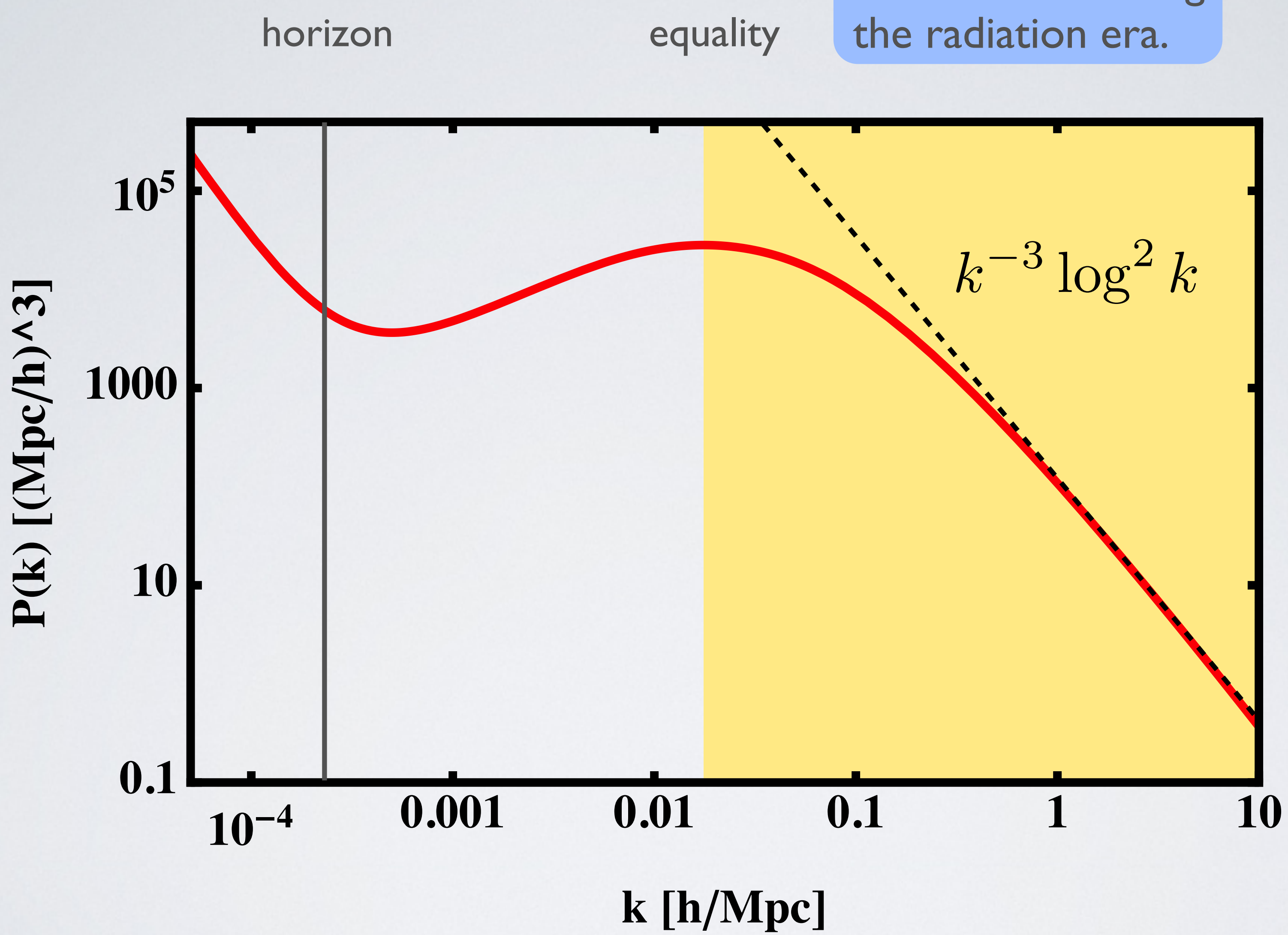
Power spectrum

Large scales: enter the horizon during the matter era.



Power spectrum

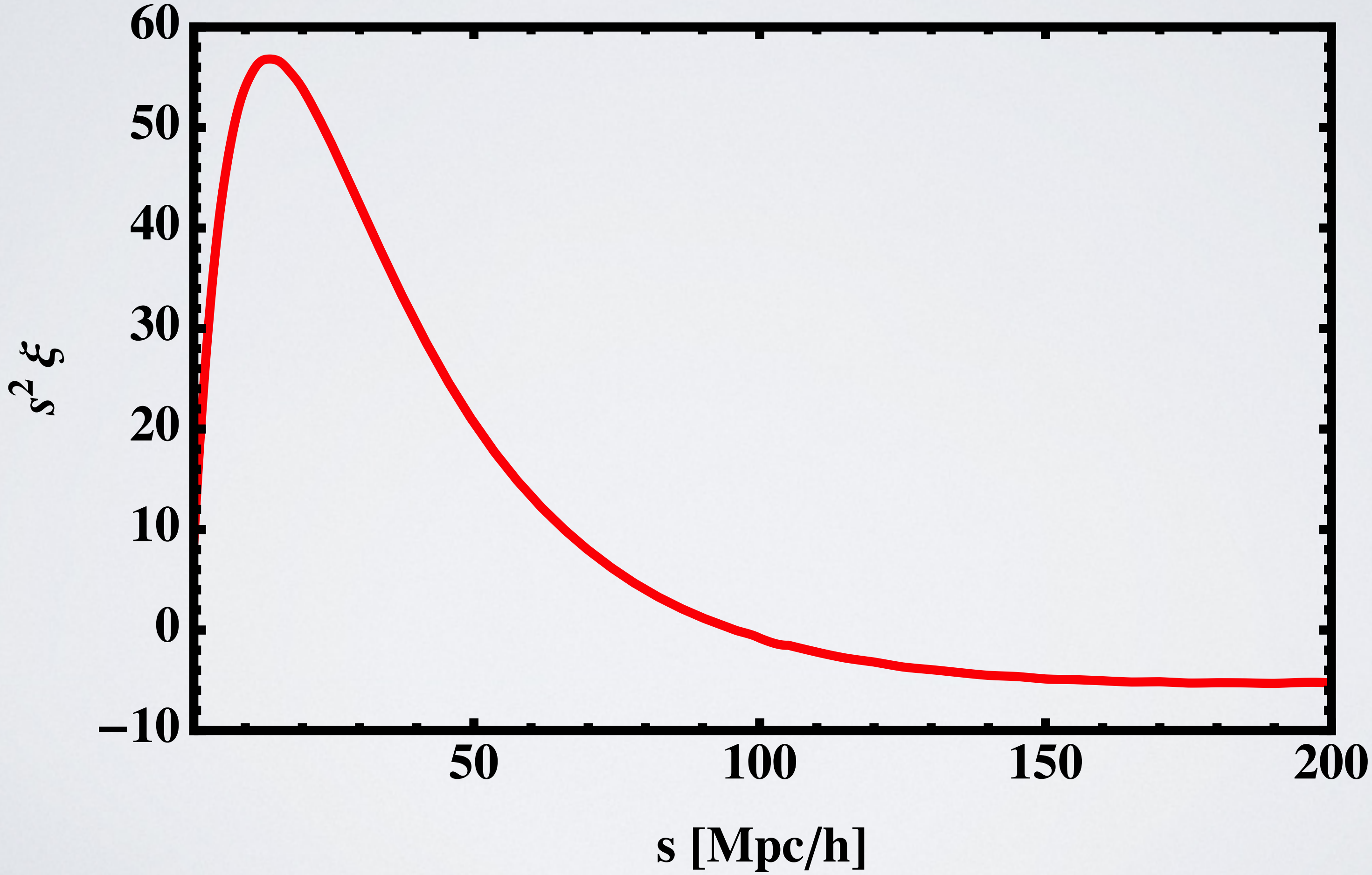
Small scales: enter the horizon during the radiation era.



Correlation function

$$\xi(|\mathbf{x}-\mathbf{x}'|,\eta_0) = \langle \delta_{dm}(\mathbf{x},\eta_0)\delta_{dm}(\mathbf{x}',\eta_0)\rangle$$

$$= \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{-i\mathbf{k}(\mathbf{x}-\mathbf{x}')} P_\delta(k,\eta_0) = \int \frac{dk\, k^2}{2\pi^2} P_\delta(k,\eta_0) j_0\big(k|\mathbf{x}-\mathbf{x}'|\big)$$



Missing elements

- Dark energy
- Redshift-space distortions
- Baryon acoustic oscillations
- Non-linearities
- Relativistic effects

Dark Energy

- From supernovae measurement we know that the Universe started **accelerating recently** at $z \sim 0.5$.
- For most of the dark matter evolution, dark energy was **negligible**.
- Dark energy affects all the observable **scales** in the **same** way, because they were all inside the horizon when dark energy started dominating.
- Dark energy will modify the **amplitude** of the power spectrum, but not its shape → the density can be expressed as:

At late time:

$$\delta_{dm}(\mathbf{k},\eta) = D_1(a) T_\delta(k) \Phi_p(\mathbf{k})$$

$\Phi_p(\mathbf{k})$ → initial conditions

$D_1(a)$ → growth rate: independent of scale

$T_\delta(k)$ → transfer function: independent of time

Dark Energy

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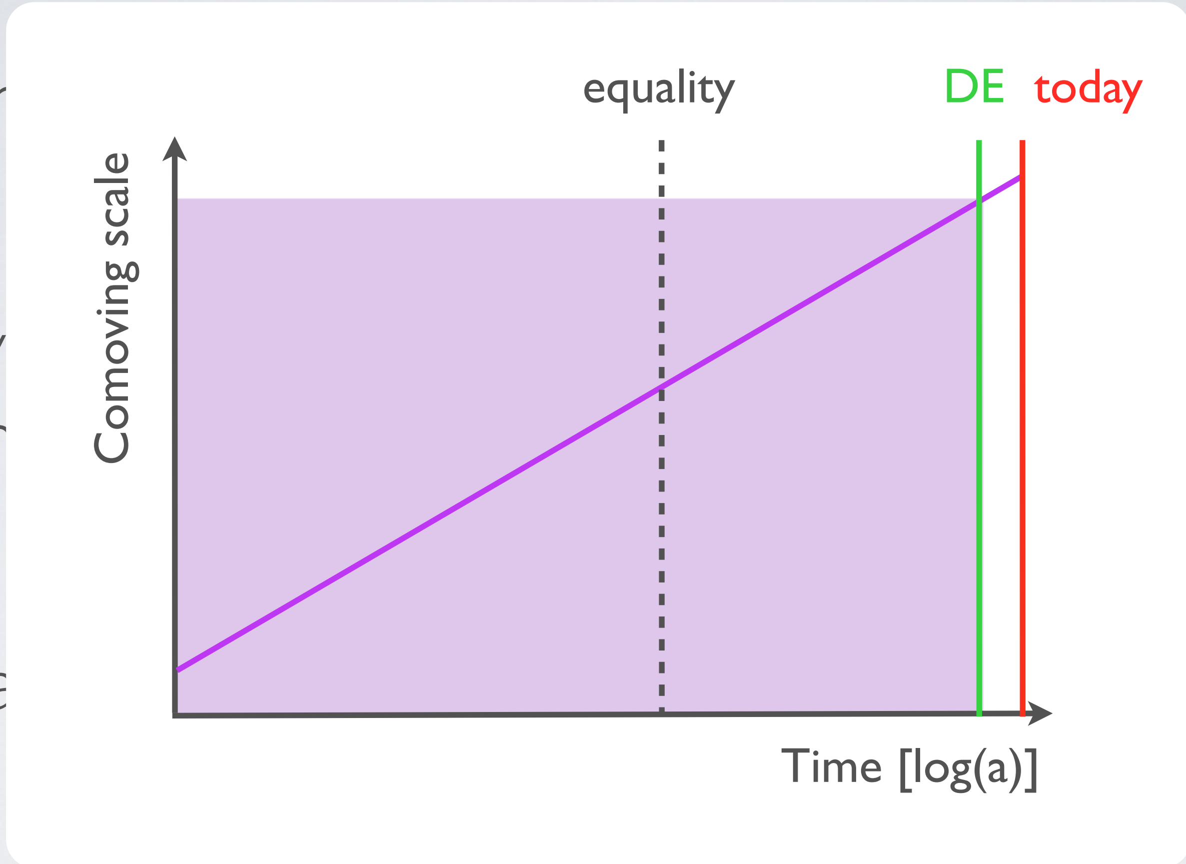

…onditions

At late time: $\delta_{dm}(\mathbf{k},\eta) = D_1(a)T_\delta(k)\Phi_p(\mathbf{k})$

growth rate:
independent of scale

transfer function:
independent of time

Dark Energy

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$T_\delta(k)$ → transfer function: independent of time

$\Phi_p(\mathbf{k})$ → initial conditions

Growth function

In a **matter dominated** universe: $D_1(a) = a$

How does this change with **dark energy**?

Intuitively we expect a **slower** growth (acceleration).

Calculation: $\delta''_{dm} + \mathcal{H}\delta'_{dm} = -k^2\Phi + 3\mathcal{H}\Phi' + 3\Phi''$

Inside the horizon: we keep only the first term.

Poisson: $-k^2\Phi = 4\pi G a^2 \bar{\rho}_{dm}\delta_{dm}$

radiation is negligible and dark energy does not cluster

$$\delta''_{dm} + \mathcal{H}\delta'_{dm} - 4\pi G a^2 \bar{\rho}_{dm}\delta_{dm} = 0$$

Growth function

Equation for the growth function:

$$D_1'' + \mathcal{H} D_1' - 4\pi G a^2 \bar{\rho}_m D_1 = 0$$

In a universe with matter and a **cosmological constant**:

$$\mathcal{H}^2 = \frac{8\pi G a^2}{3}\left(\bar{\rho}_\Lambda + \bar{\rho}_m(a)\right)$$

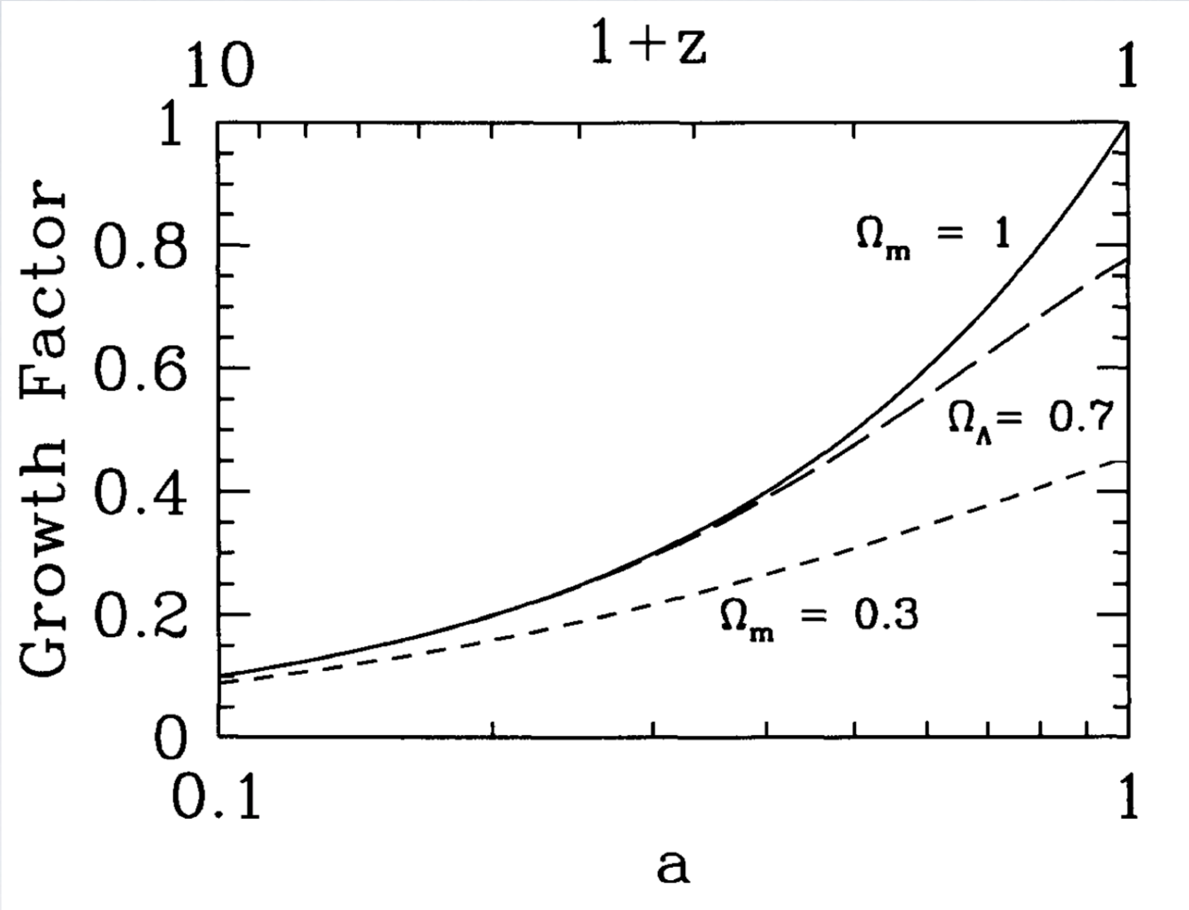
Substitution: $u = \frac{D_1}{H}$ → $\frac{d}{da}\left(\frac{du}{da}(aH)^3\right) = 0$

Growing mode:

$$D_1(a) = \frac{5\Omega_m}{2}\frac{H(a)}{H_0}\int_0^a da' \left(\frac{H_0}{a' H(a')}\right)^3$$

Result

Credit: S. Dodelson, Modern Cosmology



less growth with dark energy

At late time: $H = \text{const}$ and $D_1(a) \rightarrow \text{const}$

No growth in a universe dominated by dark energy.

Redshift-space distortions

Coordinates

- Until now we have calculated $\delta_{dm}(\mathbf{x}, \eta)$
- In Fourier space $\delta_{dm}(\mathbf{k}, \eta) = \int d^3\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}}\, \delta_{dm}(\mathbf{x}, \eta)$
- The **position** of a galaxy $\mathbf{x}$ is given by its direction and its distance $\mathbf{x} = (\mathbf{n}, r)$
- In a survey we do not measure r but we measure the **redshift** z
- We calculate the radial distance from the redshift.
- Photons travel on **null geodesics**

$$1 + z = \frac{a_0}{a} = \frac{1}{a}$$

$$dr = d\eta = \frac{d\eta}{da}\frac{da}{dz}dz = -\frac{1}{a'}\frac{1}{(1+z)^2}dz = -\frac{a}{\mathcal{H}}dz$$

Coordinates

- Radial distance $r(z) = \int_0^z dz' \frac{1}{(1+z')\mathcal{H}(z')}$
- The relation depends on the **cosmology** (from SNe, CMB).
- **Problem**: the above relation between redshift and radial distance is only correct in a **homogeneous** universe, where the redshift is entirely due to the expansion of the universe:

$$1 + z = \frac{1}{a}$$

- In a universe with **fluctuations**, the redshift is affected by other **effects**.

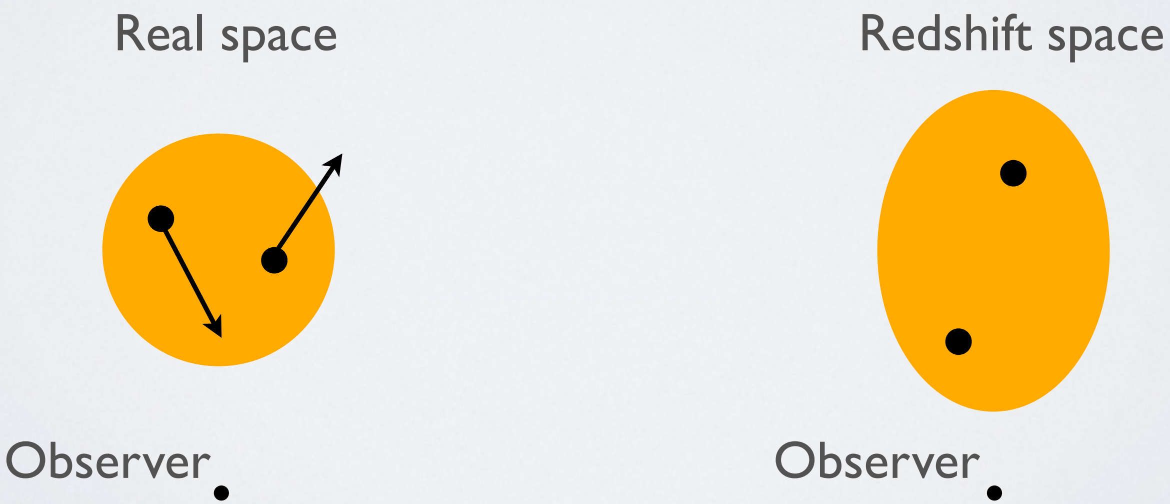
Doppler effect

- The **motion** of galaxies with respect to us induces a **Doppler** shift.
- Homogeneous universe: all galaxies move with the Hubble flow.
- Inhomogeneous universe: galaxies are attracted towards over-dense regions.
- The **positions** of the galaxies on the map are **shifted**.
- Consequence: this changes the **observed large-scale structure**, e.g. the shape over-densities.

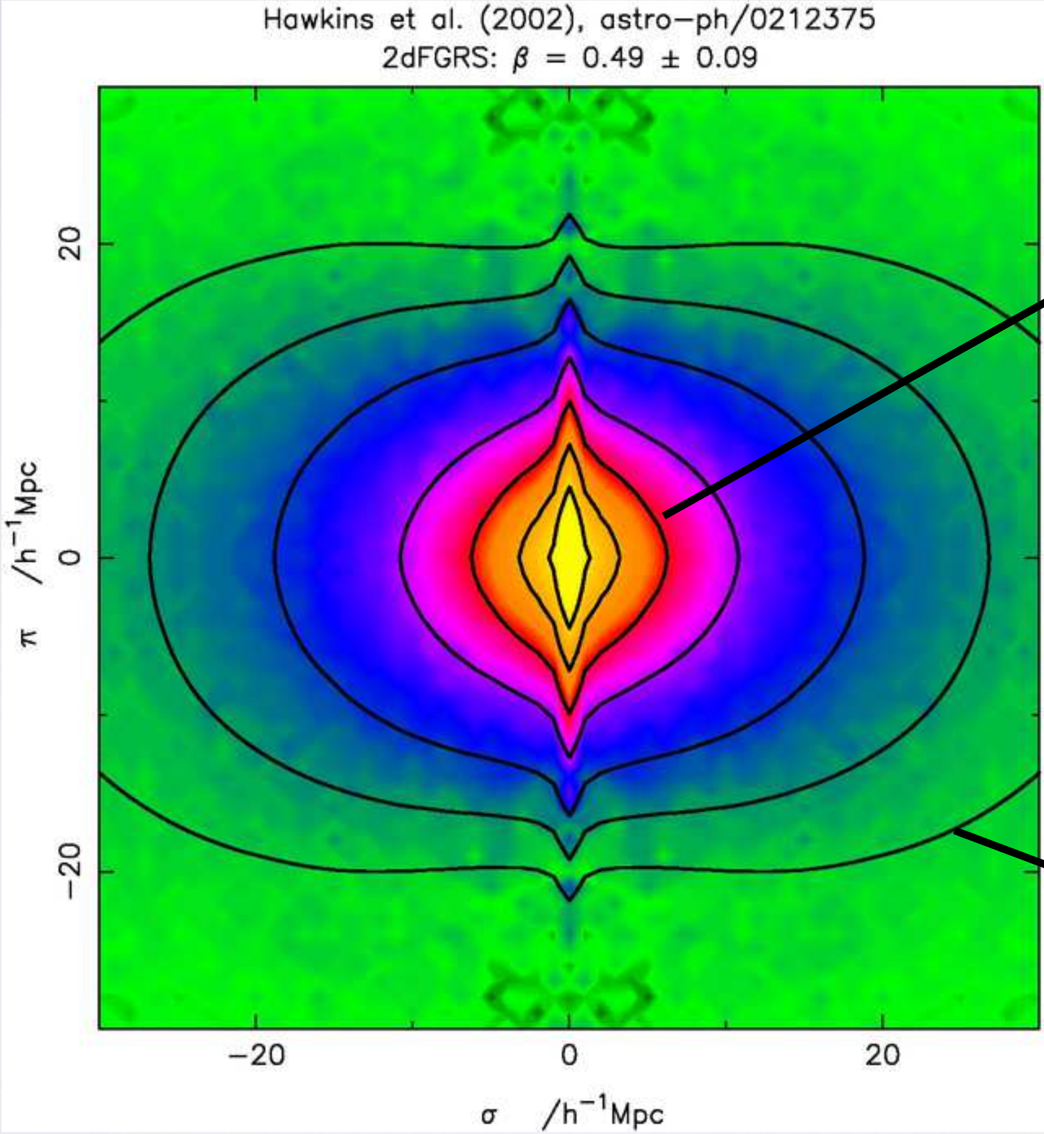
Distortions

Linear regime: over-densities are **squeezed** along the line of sight.

Non-linear regime: virialised objects (clusters) are **elongated**, fingers of god effect.



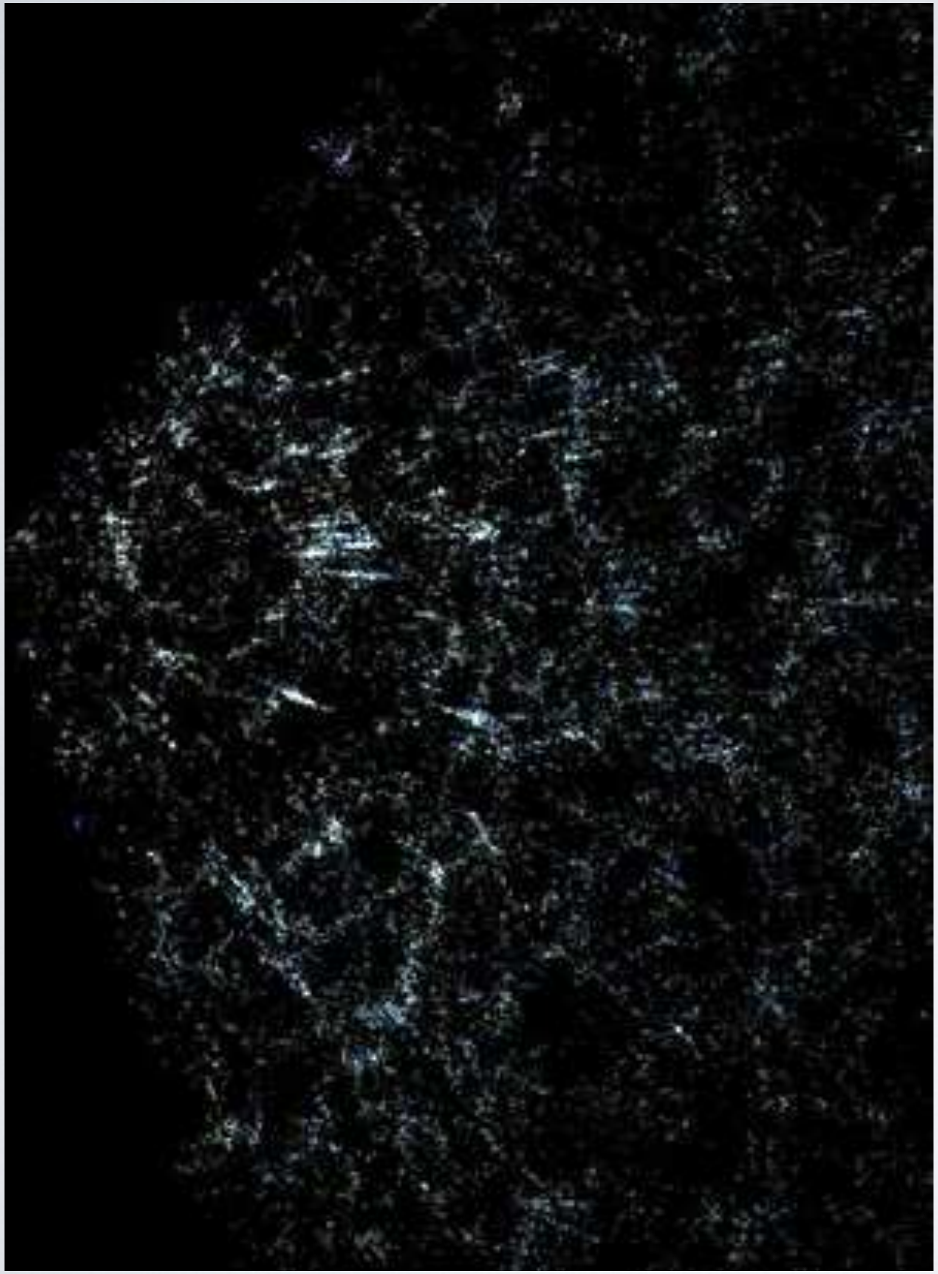
Contours



non-linear elongation

linear squeeze

Fingers of god



Fingers of God in a portion of the Sloan Digital Sky Survey.
Image from the Cosmus Open Source Science Outreach project.

The **non-linear** effect can be seen by eyes.

The **linear** redshift-space distortion is statistically detectable in the correlation function.

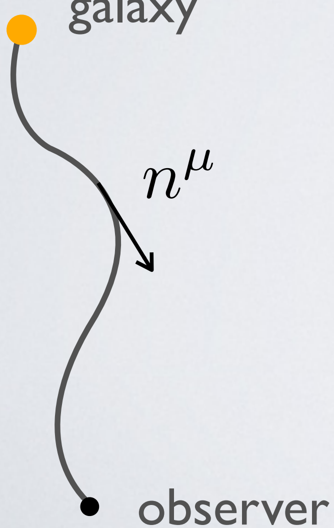
Redshift

Effect of peculiar velocity on the redshift: $1+z=\frac{\nu_S}{\nu_O}=\frac{E_S}{E_O}$

How does the **energy change** between source and observer?

Photons travel on **null geodesics** determined by the metric.

$$ds^2=-a^2\Big[\big(1+2\Psi\big)d\eta^2+\big(1-2\Phi\big)\delta_{ij}dx^idx^j\Big]$$



geodesic equation: $\frac{dn^\mu}{d\lambda}+\Gamma^\mu_{\alpha\beta}n^\alpha n^\beta=0$

null geodesic: $n^\mu n_\mu=0$

background: $n^\mu=\frac{1}{a^2}(1,-\mathbf{n})$

fluctuation: $n^\mu(\Psi,\Phi)$

Redshift

The energy is the projection of n^μ onto the velocity $u^\mu = \frac{1}{a}(u^0, \mathbf{v})$

proper time   peculiar velocity

$$E = -n^\mu u_\mu \qquad\qquad 1 + z = \frac{E_S}{E_O}$$

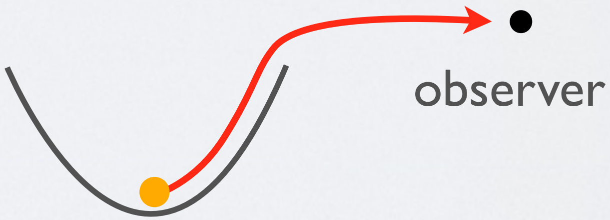
Background: $1 + \bar{z} = \frac{a_O}{a_S}$

Fluctuations:

$$1 + z = \frac{a_O}{a_S}\left[1 + \mathbf{v}_S \cdot \mathbf{n} - \mathbf{v}_O \cdot \mathbf{n} + \Psi_O - \Psi_S - \int_0^{r_S} dr(\dot{\Phi} + \dot{\Psi})\right]$$

Doppler   Gravitational redshift   Integrated Sachs-Wolfe

Gravitational redshift:

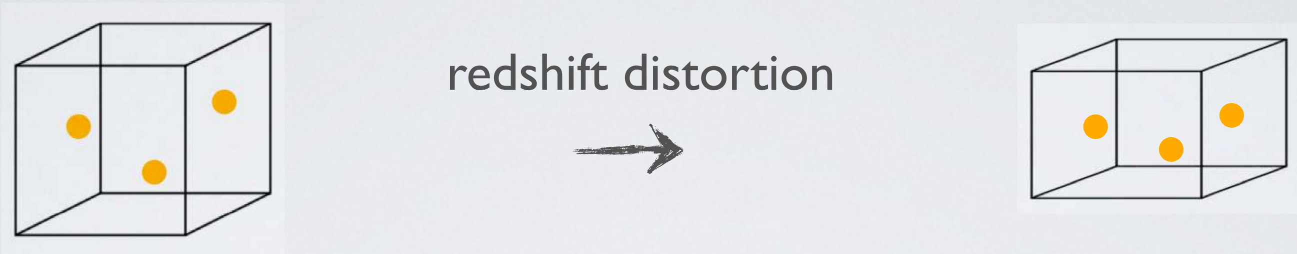


ISW: integrated along the trajectory. Sensitive to dark energy.

Galaxy distribution

How do the redshift fluctuations affect the observation of δ ?

We extract the number of galaxies per **volume element**.



Number of galaxies is **conserved**: $\rho(\mathbf{x}_{\mathrm{obs}})\, d^3\mathbf{x}_{\mathrm{obs}} = \rho(\mathbf{x})\, d^3\mathbf{x}$

$$\bar{\rho}\big(1+\delta_{\mathrm{obs}}\big)\, d^3\mathbf{x}_{\mathrm{obs}} = \bar{\rho}\big(1+\delta\big)\, d^3\mathbf{x}$$

The change in δ_{obs} is due to the change from $\mathbf{x}$ to $\mathbf{x}_{\mathrm{obs}}$

Only the **radial coordinate** is affected by redshift perturbations

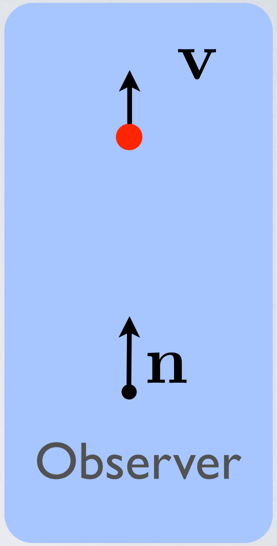
$$r_{\mathrm{obs}} = r(z) = r(\bar{z}+\delta z) \simeq r(\bar{z}) + \frac{\partial r}{\partial \bar{z}}\,\delta z$$

Galaxy over-density

$$r_{\rm obs} = r + \frac{\partial r}{\partial \bar z}\,\delta z \qquad\qquad \frac{\partial r}{\partial \bar z} = \frac{1}{(1+\bar z)\mathcal{H}}$$

We keep only the Doppler contribution: $r_{\rm obs} = r + \frac{1}{\mathcal{H}}\mathbf{v}\cdot\mathbf{n}$

Jacobian: $$\frac{\partial r_{\rm obs}}{\partial r} = 1 + \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n}) + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\mathbf{v}\cdot\mathbf{n}$$



Neglecting the second term:

$$(1+\delta_{\rm obs})\left[1 + \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n})\right]d^3\mathbf{x} = (1+\delta)d^3\mathbf{x}$$

$$\boxed{\delta_{\rm obs} = \delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n})}$$

Kaiser (1987)

Galaxy over-density

$$r_{\text{obs}} = r + \frac{\partial r}{\partial \bar{z}} \delta z \qquad \frac{\partial r}{\partial \bar{z}} = \frac{1}{(1+\bar{z})\mathcal{H}}$$

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Neglecting the second term:

$$(1+\delta_{\text{obs}}) \left[1 + \frac{1}{\mathcal{H}} \partial_r (\mathbf{v} \cdot \mathbf{n})\right] d^3\mathbf{x} =$$

We see a distorted distribution of galaxies because we are not able to measure distances directly. How problematic is that?

$$\delta_{\text{obs}} = \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{v} \cdot \mathbf{n})$$

Kaiser (1987)

Interest of redshift distortions

Redshift distortions provides an opportunity to measure **peculiar velocities**. Galaxies move according to dark matter inhomogeneities → another way of mapping the matter distribution.

We already know the peculiar velocities from **Euler** equation:

$$\delta' = kv$$

- We want to **test** Euler equation (interacting DE and DM)
- Velocities measure directly the **evolution** of the density. More sensitive to modified gravity.
- Peculiar velocities are not sensitive to **bias**:

$$\delta = b \cdot \delta_{dm} \qquad \text{but} \qquad v = v_{dm}$$

Anisotropy

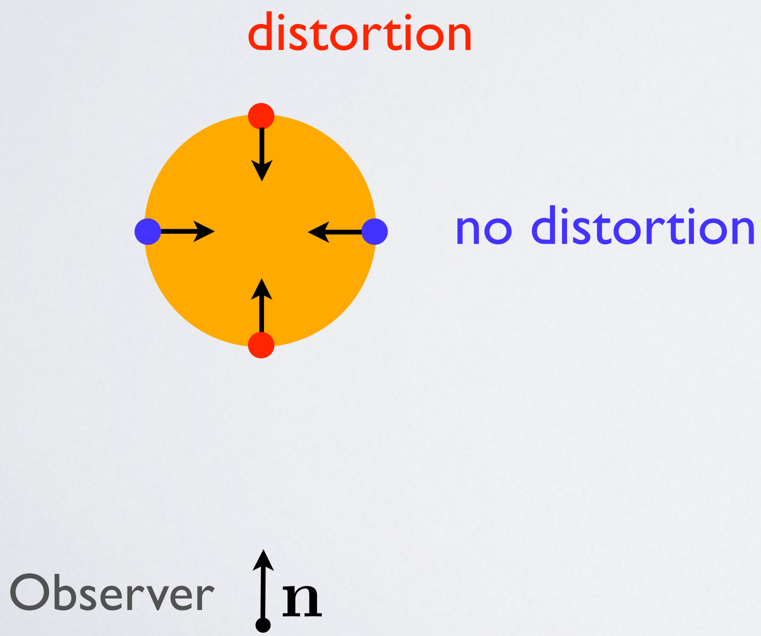
How do we measure redshift distortions and separate velocities from density?

The velocity part is **anisotropic**

line of sight

$$\delta_{\rm obs} = \delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n})$$

We expect differences along and transverse to the **line-of-sight.**



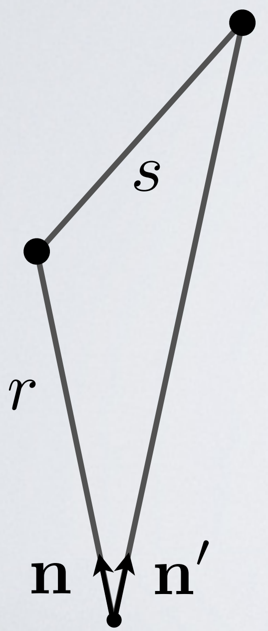
We can detect this anisotropy **statistically** in the correlation function.

Anisotropy

Two-point correlation function:

$$\xi = \left\langle \left(\delta(\mathbf{x},\eta) - \frac{1}{\mathcal{H}}\partial_r \mathbf{v}(\mathbf{x},\eta)\cdot\mathbf{n}\right)\left(\delta(\mathbf{x}',\eta') - \frac{1}{\mathcal{H}}\partial_{r'} \mathbf{v}(\mathbf{x}',\eta')\cdot\mathbf{n}'\right)\right\rangle$$

Without distortion: $\xi(s,r)$

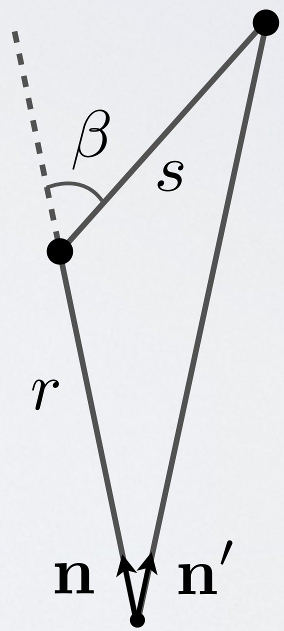


Observer

Depends on:

- separation
- distance of the pair

With distortion: $\xi(s,r,\beta)$



Observer

Additional dependence on orientation:

max signal: $\beta = 0,\ \pi$

min signal: $\beta = \frac{\pi}{2}$

Anisotropy

Two-point correlation function:

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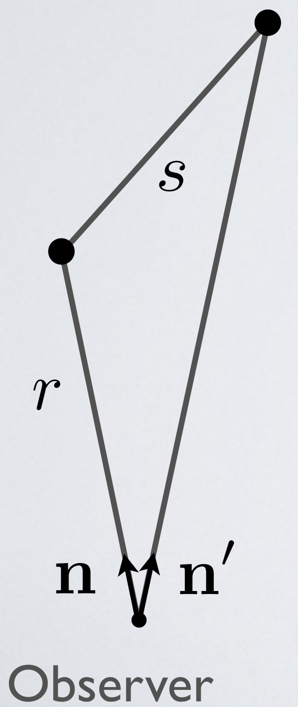

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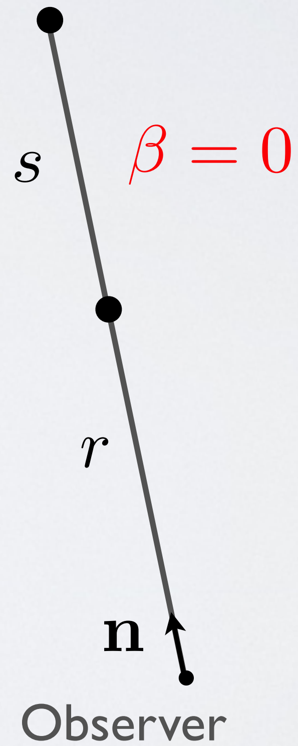

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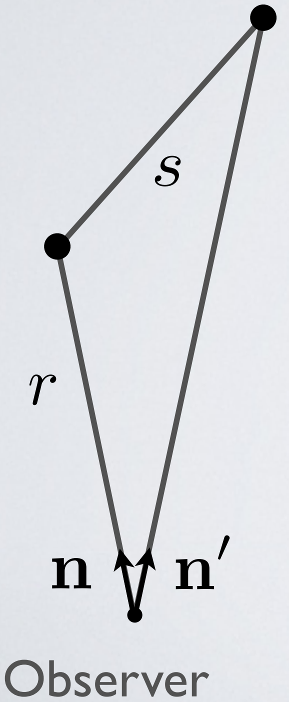

Depends on:

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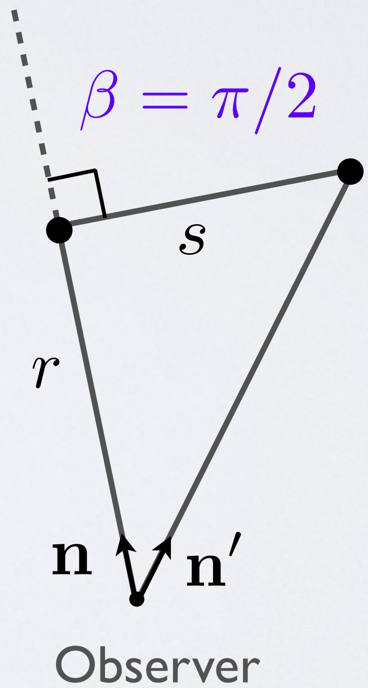
- distance of the pair

With distortion: $\xi(s, r, \beta)$



Additional dependence on orientation:

max signal: $\beta = 0,\ \pi$

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Derivation

$$\xi = \left\langle \left(\delta(\mathbf{x},\eta) - \frac{1}{\mathcal{H}}\partial_r \mathbf{v}(\mathbf{x},\eta)\cdot\mathbf{n}\right)\left(\delta(\mathbf{x}',\eta') - \frac{1}{\mathcal{H}}\partial_{r'} \mathbf{v}(\mathbf{x}',\eta')\cdot\mathbf{n}'\right)\right\rangle$$

- Fourier transform $\delta(\mathbf{x},\eta) \to \delta(\mathbf{k},\eta) \quad \mathbf{v}(\mathbf{x},\eta) \to \mathbf{v}(\mathbf{k},\eta) = i\,\hat{\mathbf{k}}\,v(\mathbf{k},\eta)$
- Euler equation $v(\mathbf{k},\eta) = \frac{1}{k}\,\delta'(\mathbf{k},\eta)$
- Relate density to primordial potential $\delta(\mathbf{k},\eta) = D_1(a)T_\delta(k)\Phi_p(\mathbf{k})$

$$\frac{1}{\mathcal{H}}\partial_r\,\mathbf{v}(\mathbf{x},\eta)\cdot\mathbf{n} \;\to\; \frac{k}{\mathcal{H}}(\hat{\mathbf{k}}\cdot\mathbf{n})^2\,v(\mathbf{k},\eta) = (\hat{\mathbf{k}}\cdot\mathbf{n})^2\frac{D_1'(a)}{\mathcal{H}}T_\delta(k)\Phi_p(\mathbf{k})$$

$$\frac{d}{d\eta}D_1(a) = \mathcal{H}\,a\frac{d}{da}D_1(a)$$

$$f = \frac{a}{D_1}\frac{d}{da}D_1$$ evolution

anisotropy

$$\delta(\mathbf{x},\eta) - \frac{1}{\mathcal{H}}\partial_r\mathbf{v}(\mathbf{x},\eta)\cdot\mathbf{n} \;\to\; \left(1 + (\hat{\mathbf{k}}\cdot\mathbf{n})^2 f\right)D_1(a)T_\delta(k)\Phi_p(\mathbf{k})$$

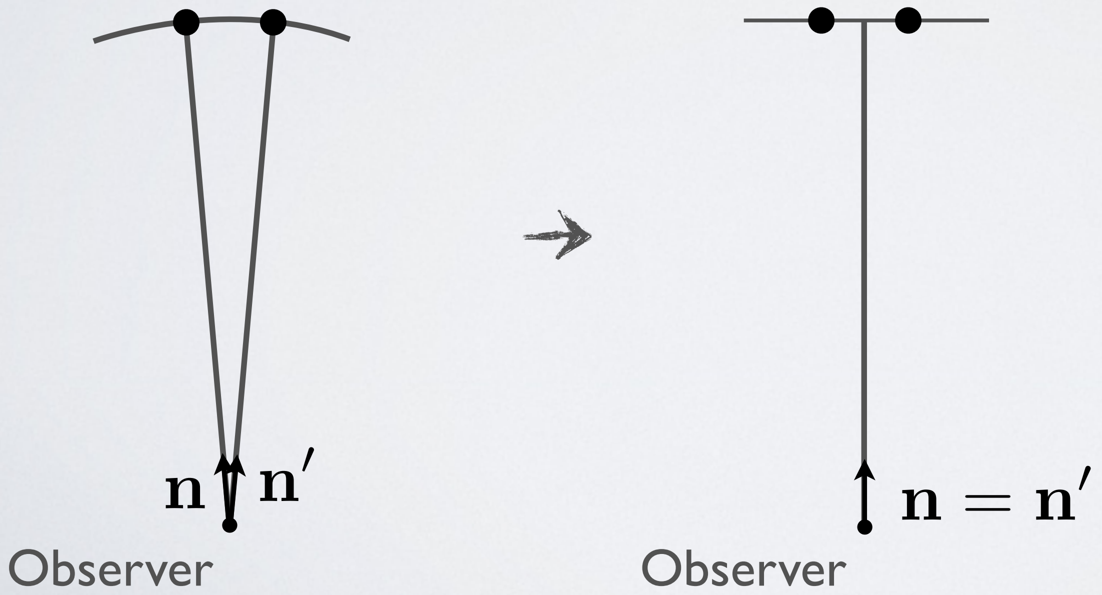
Derivation

$$\xi = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} e^{-i(\mathbf{k}\cdot\mathbf{x}+\mathbf{k}'\cdot\mathbf{x}')} \left(1 + f(\eta)(\hat{\mathbf{k}}\cdot\mathbf{n})^2\right)\left(1 + f(\eta')(\hat{\mathbf{k}}'\cdot\mathbf{n}')^2\right)$$

$$\times\, D_1(a)\, D_1(a')\, T_\delta(k)\, T_\delta(k')\, \langle \Phi_p(\mathbf{k})\Phi_p(\mathbf{k}')\rangle$$

known initial conditions $\sim \frac{1}{k^3}\delta_D(\mathbf{k}+\mathbf{k}')$

Simplification: **distant observer** approximation: $\mathbf{n} = \mathbf{n}'$



the sky is approximated as flat → there is only one line-of-sight for each pair.

Derivation

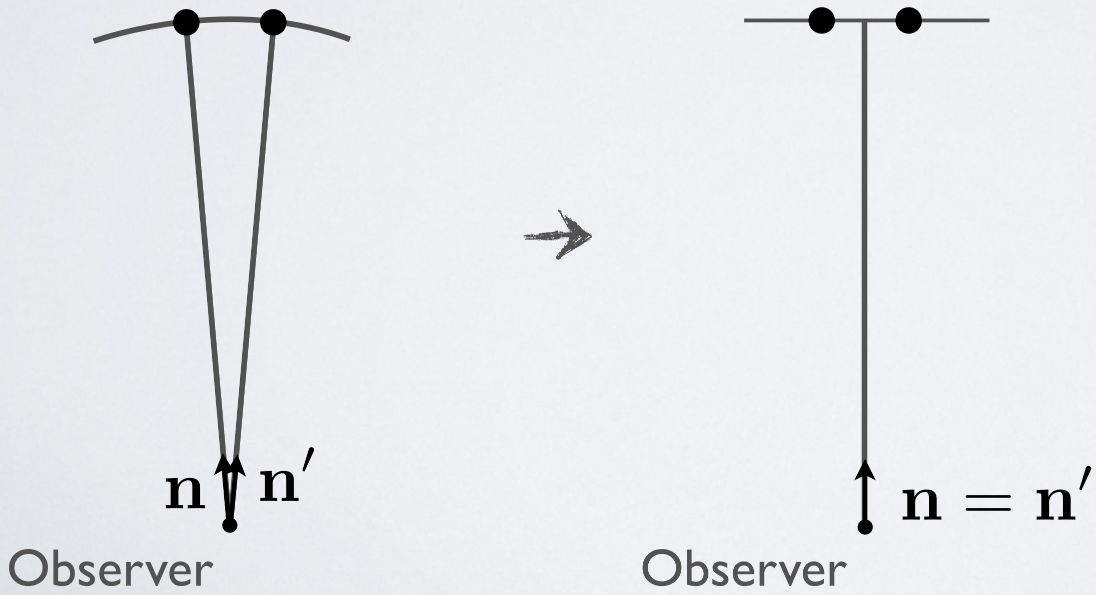
depends on the direction of $\mathbf{k}$

$$\xi = \int \frac{d^3\mathbf{k}}{(2\pi)^6} e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \left(1 + f(\eta)(\hat{\mathbf{k}}\cdot\mathbf{n})^2\right)\left(1 + f(\eta')(\hat{\mathbf{k}}\cdot\mathbf{n}')^2\right)$$

$$\times\, D_1(a)\, D_1(a')\, T_\delta^2(k)\, \langle \Phi_p(\mathbf{k})\Phi_p(\mathbf{k}')\rangle$$

known initial conditions $\sim \dfrac{1}{k^3}\delta_D(\mathbf{k}+\mathbf{k}')$

Simplification: **distant observer** approximation: $\mathbf{n} = \mathbf{n}'$



the sky is approximated as flat → there is only one line-of-sight for each pair.

Result

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s)P_2(\cos\beta) + \frac{8f^2}{35}\mu_4(s)P_4(\cos\beta) \right\}$$

Hamilton (1992)

$$\mu_\ell(s) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta^2(k)\, j_\ell(k\cdot s)$$

sets the shape of the correlation as a function of separation.

other terms: ♦ cross-terms **density-velocity** proportional to f

♦ **velocity-velocity** terms proportional to f^2

with $f = \frac{a}{D_1}\frac{d}{da}D_1$

Result

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right) \mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right) \mu_2(s) P_2(\cos\beta) + \frac{8f^2}{35} \mu_4(s) P_4(\cos\beta) \right\}$$

growth function

Hamilton (1992)

primordial amplitude

$$\mu_0(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left(\frac{k}{H_0}\right)^{n_s - 1} T_\delta^2(k)\, j_0(k \cdot s)$$

sets the shape of the correlation as a function of separation.

transfer function

separation

other terms: ♦ cross-terms **density-velocity** proportional to f

♦ **velocity-velocity** terms proportional to f^2

with $f = \frac{a}{D_1} \frac{d}{da} D_1$

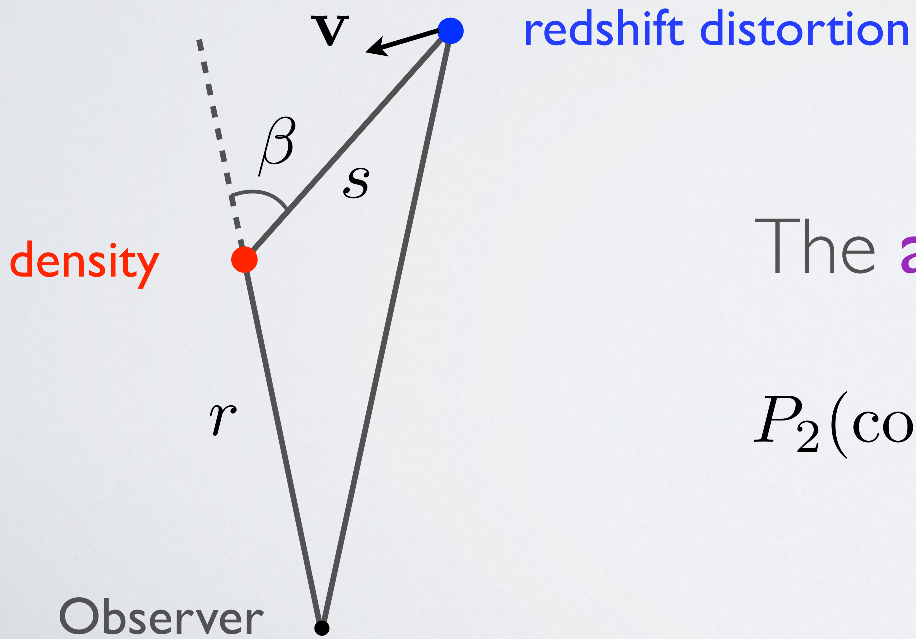
Result

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right) \mu_2(s) P_2(\cos\beta) + \frac{8f^2}{35}\mu_4(s)P_4(\cos\beta) \right\}$$

Hamilton (1992)

$$\mu_2(s) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta^2(k)\, j_2(k\cdot s)$$

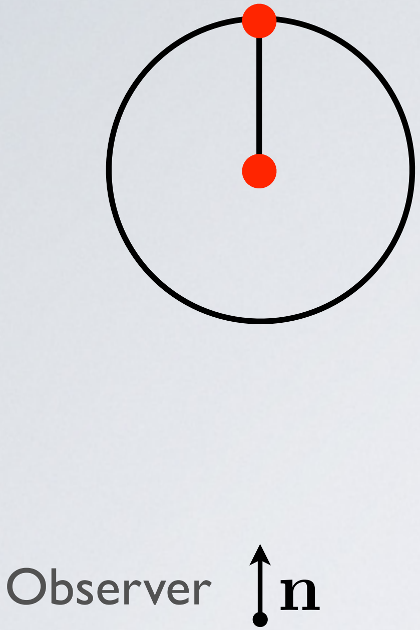
slightly different dependence in separation than the density

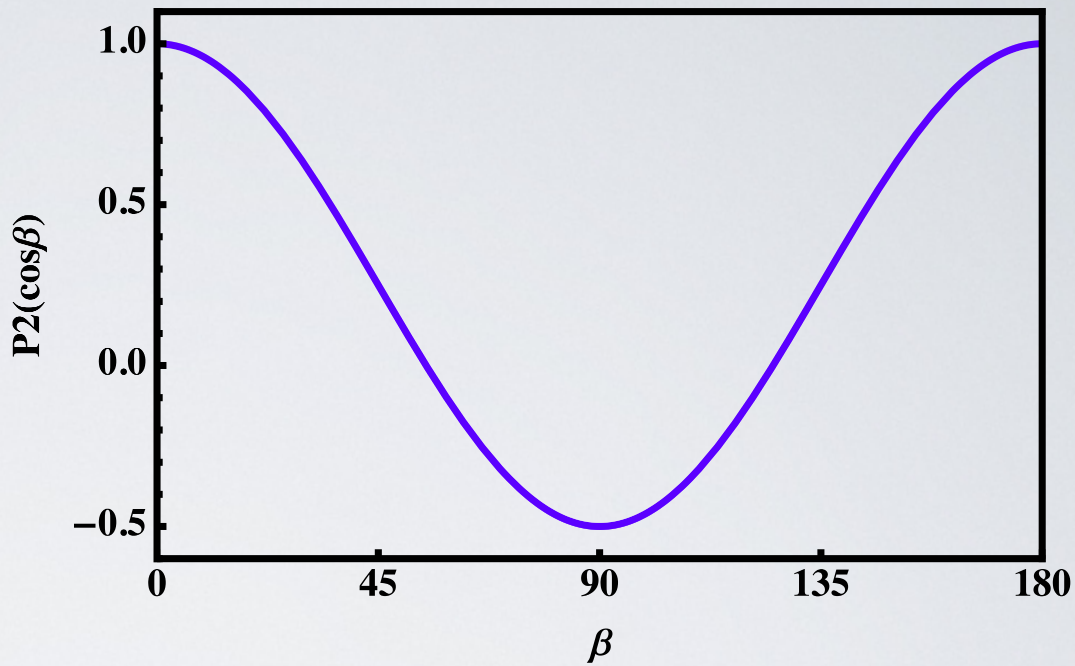


The **angular dependence** is given by:

$$P_2(\cos\beta) = \frac{3}{2}\cos^2\beta - \frac{1}{2}$$

Quadrupole dependence

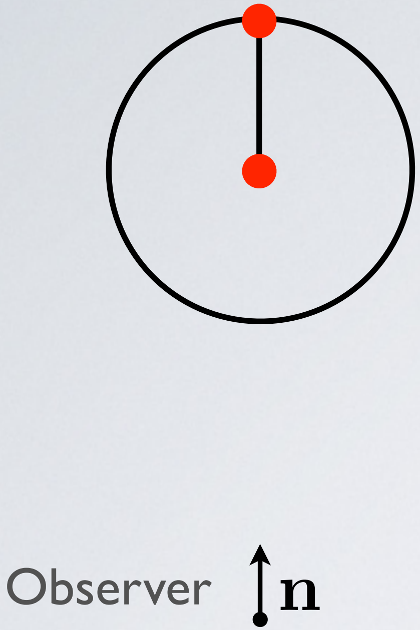


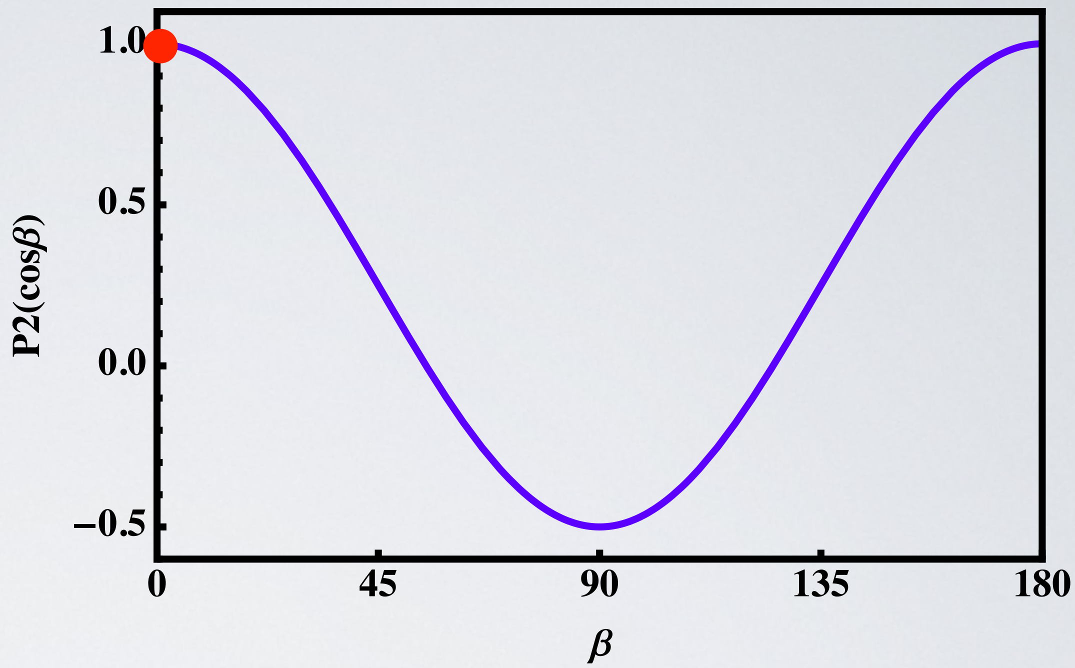


- The amplitude of the correlation function is modulated by $P_2(\cos\beta)$

- The quadrupole is **negative**: $-\dfrac{4f}{3}$

Quadrupole dependence





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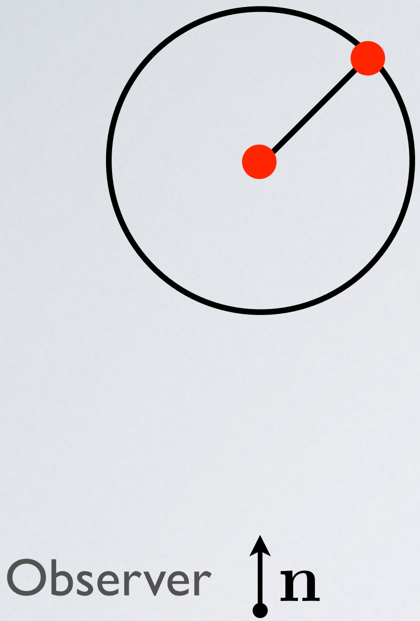
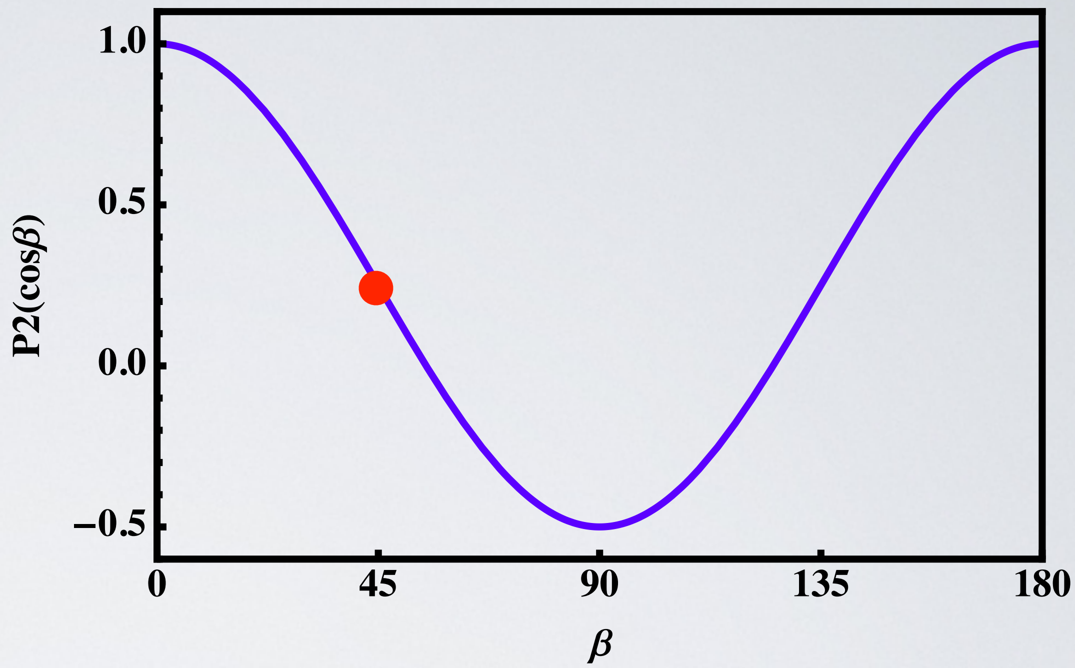
Quadrupole dependence





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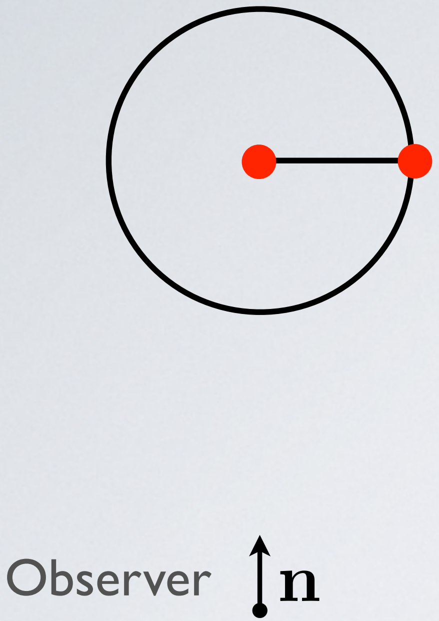
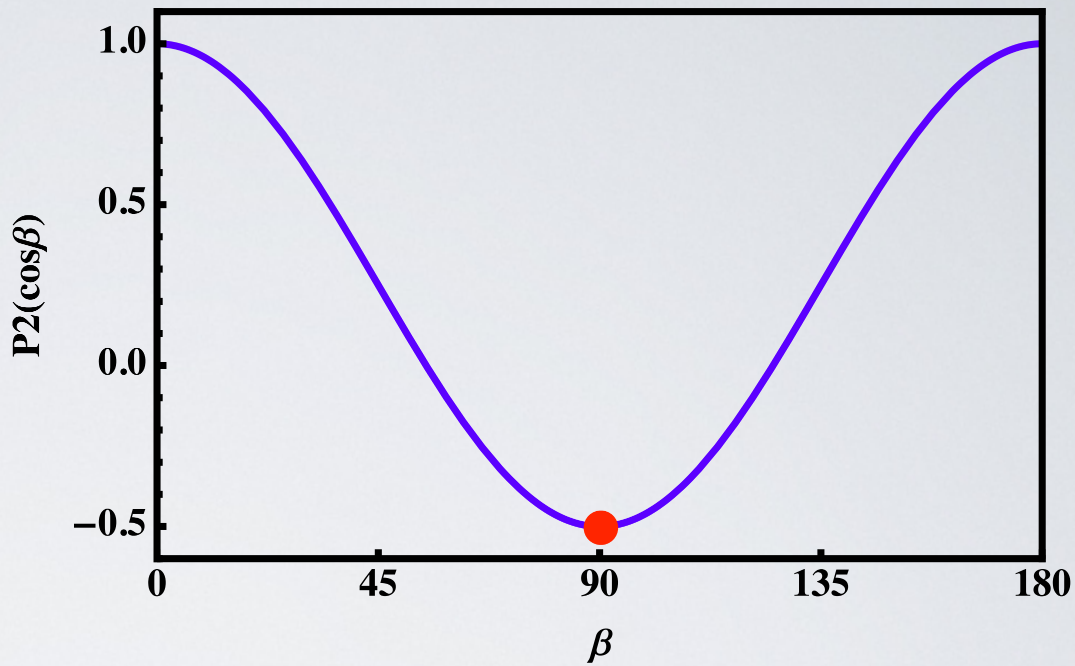
Quadrupole dependence





- The amplitude of the correlation function is modulated by $P_2(\cos\beta)$
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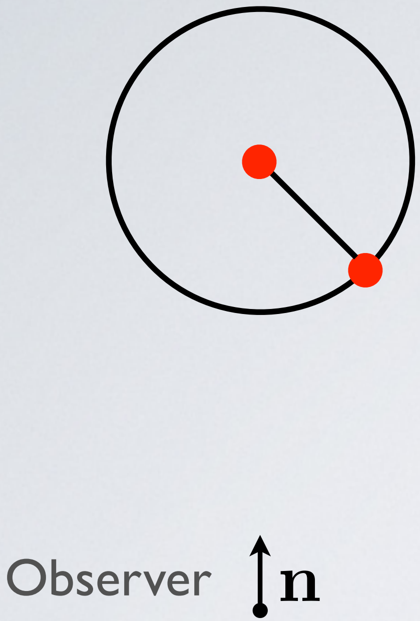
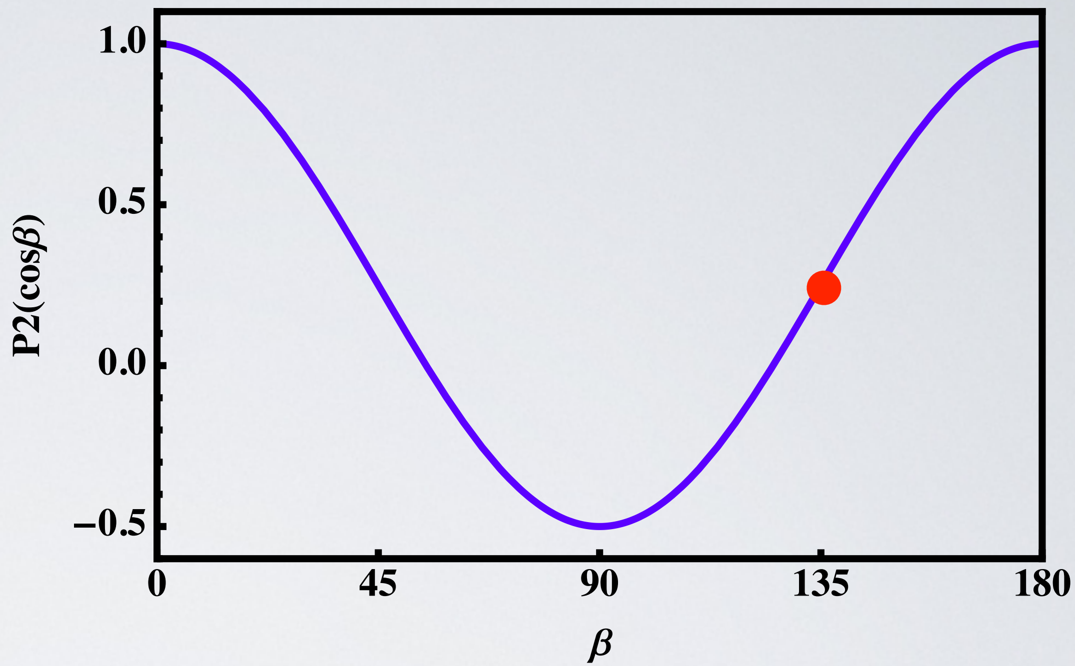
Quadrupole dependence





- The amplitude of the correlation function is modulated by $P_2(\cos\beta)$
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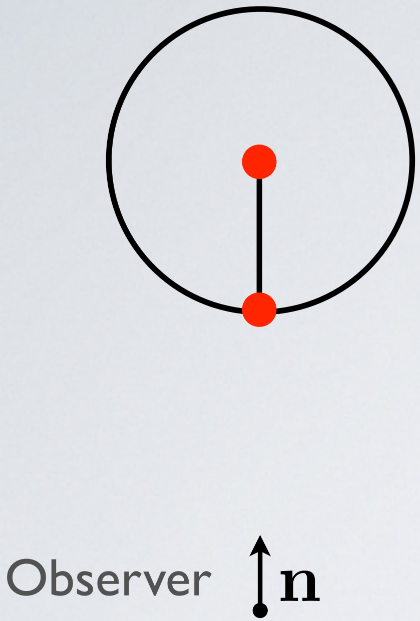
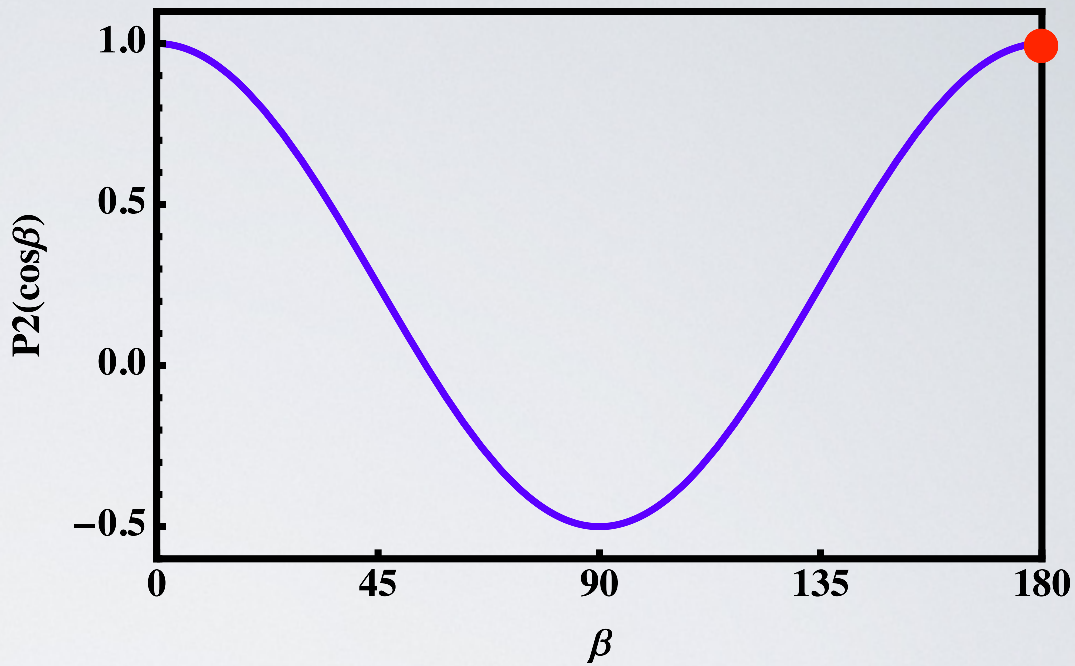
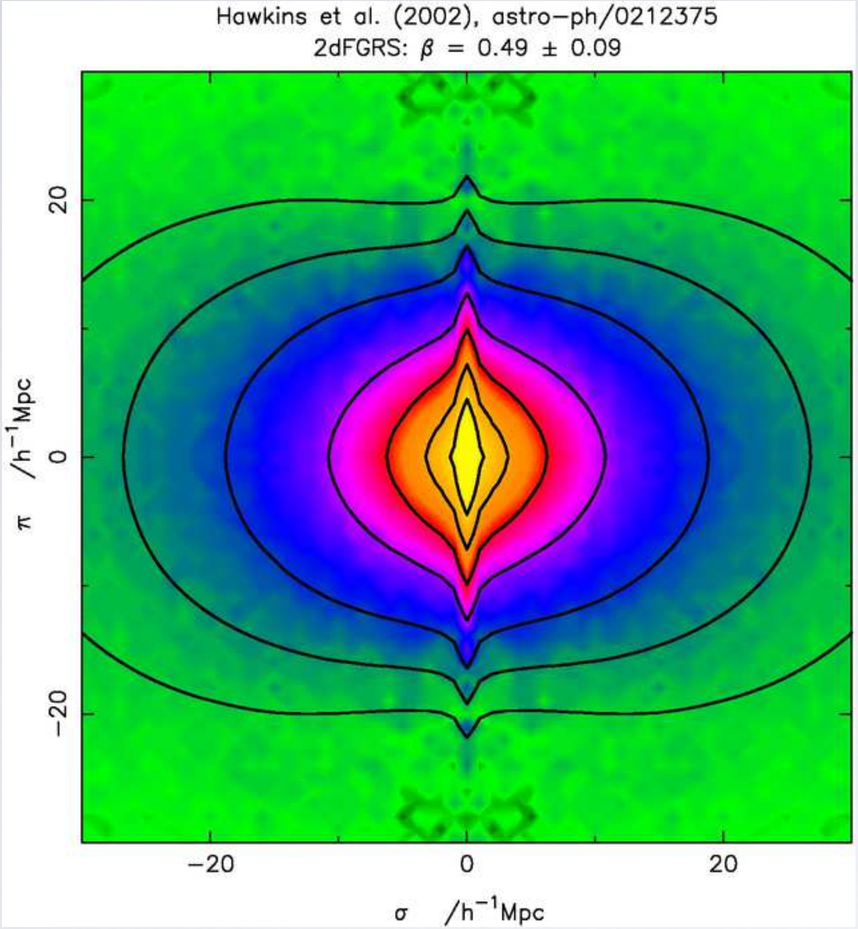
Quadrupole dependence





- The amplitude of the correlation function is modulated by $P_2(\cos\beta)$
- The quadrupole is **negative**: $-\frac{4f}{3}$

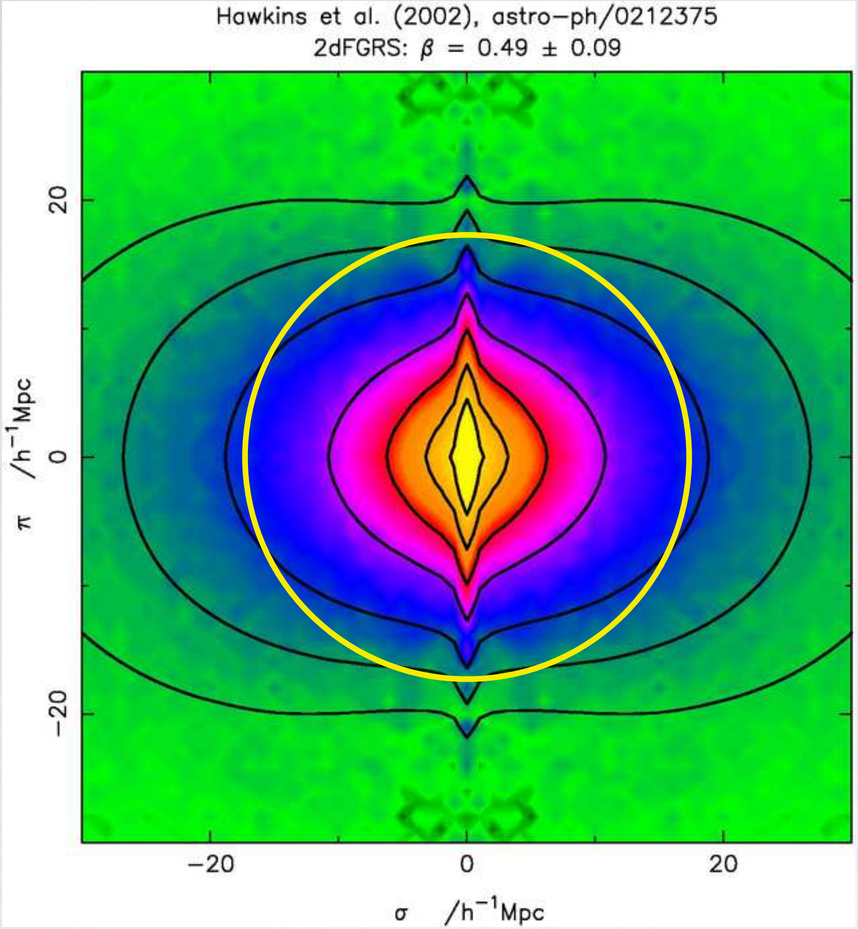
Negative quadrupole



Redshift distortions increase the **gradient** along the line-of-sight.

At a given separation, the correlation is **stronger transverse** to the line-of-sight than along the line-of-sight → negative quadrupole.

Negative quadrupole



Redshift distortions increase the **gradient** along the line-of-sight.

At a given separation, the correlation is **stronger transverse** to the line-of-sight than along the line-of-sight → negative quadrupole.

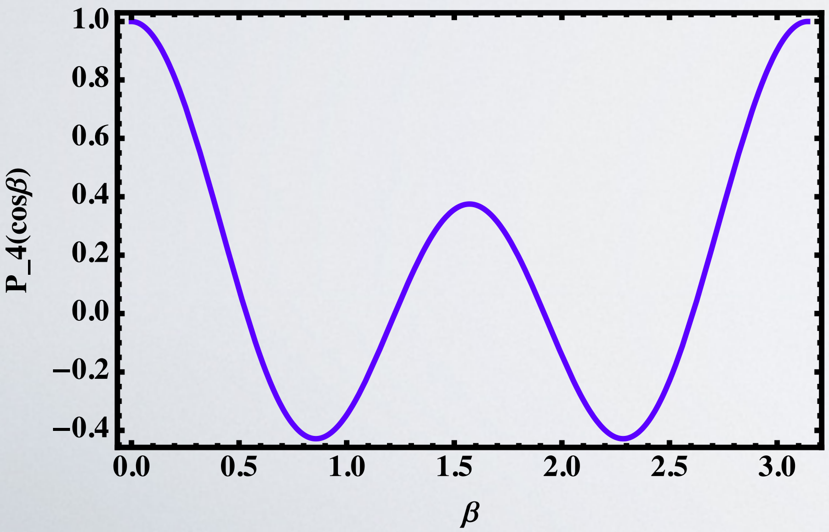
Hexadecapole dependence

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s)P_2(\cos\beta) + \frac{8f^2}{35}\mu_4(s)P_4(\cos\beta)\right\}$$

Hamilton (1992)

Other terms: velocity-velocity correlations contribute to the quadrupole and generate an **hexadecapole**.

$$P_4(\cos\beta) = \frac{1}{8}\left[35\cos^4\beta - 30\cos^2\beta + 3\right]$$



Maximum at $\beta = 0$ and π

Velocity-density decreases monotonically.

Velocity-velocity have a complicated structure due to a combination of $\cos^2\beta$

Multipoles extraction

How can we **separate** redshift distortions from density?

We can use the particular **angular dependence** of the terms.

We **average** over all orientations: $\frac{1}{2}\int_{-1}^{1} d\mu\, \xi(s, r, \mu)$

$$\int_{-1}^{1} d\mu\, P_2(\mu) = 0 \quad \text{and} \quad \int_{-1}^{1} d\mu\, P_4(\mu) = 0$$

→ extract the monopole

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s)P_2(\cos\beta) + \frac{8f^2}{35}\mu_4(s)P_4(\cos\beta)\right\}$$

Multipoles extraction

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s)P_2(\cos\beta) + \frac{8f^2}{35}\mu_4(s)P_4(\cos\beta)\right\}$$

Hamilton (1992)

- To extract the **quadrupole** we weight by $P_2(\mu)$

$$\frac{5}{2}\int_{-1}^{1} d\mu\, \xi(s,r,\mu)P_2(\mu) = -D_1^2\left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s)$$

- To extract the **hexadecapole** we weight by $P_4(\mu)$

$$\frac{9}{2}\int_{-1}^{1} d\mu\, \xi(s,r,\mu)P_4(\mu) = D_1^2\,\frac{8f^2}{35}\,\mu_4(s)$$

Measure f

Bias

Fluctuations in the number of **galaxies** are biased with respect to the **dark matter** fluctuations: $\delta = b \cdot \delta_{dm}$

$$\xi = D_1^2 \left\{ \left(b^2 + \frac{2bf}{3} + \frac{f^2}{5} \right) \mu_0(s) - \left(\frac{4bf}{3} + \frac{4f^2}{7} \right) \mu_2(s) P_2(\cos\beta) + \frac{8f^2}{35} \mu_4(s) P_4(\cos\beta) \right\}$$

The monopole and quadrupole are affected by bias, but the hexadecapole is not. This reflects the fact that the **velocities** are **not biased** $v = v_{dm}$

By measuring all multipoles we can measure both b and f

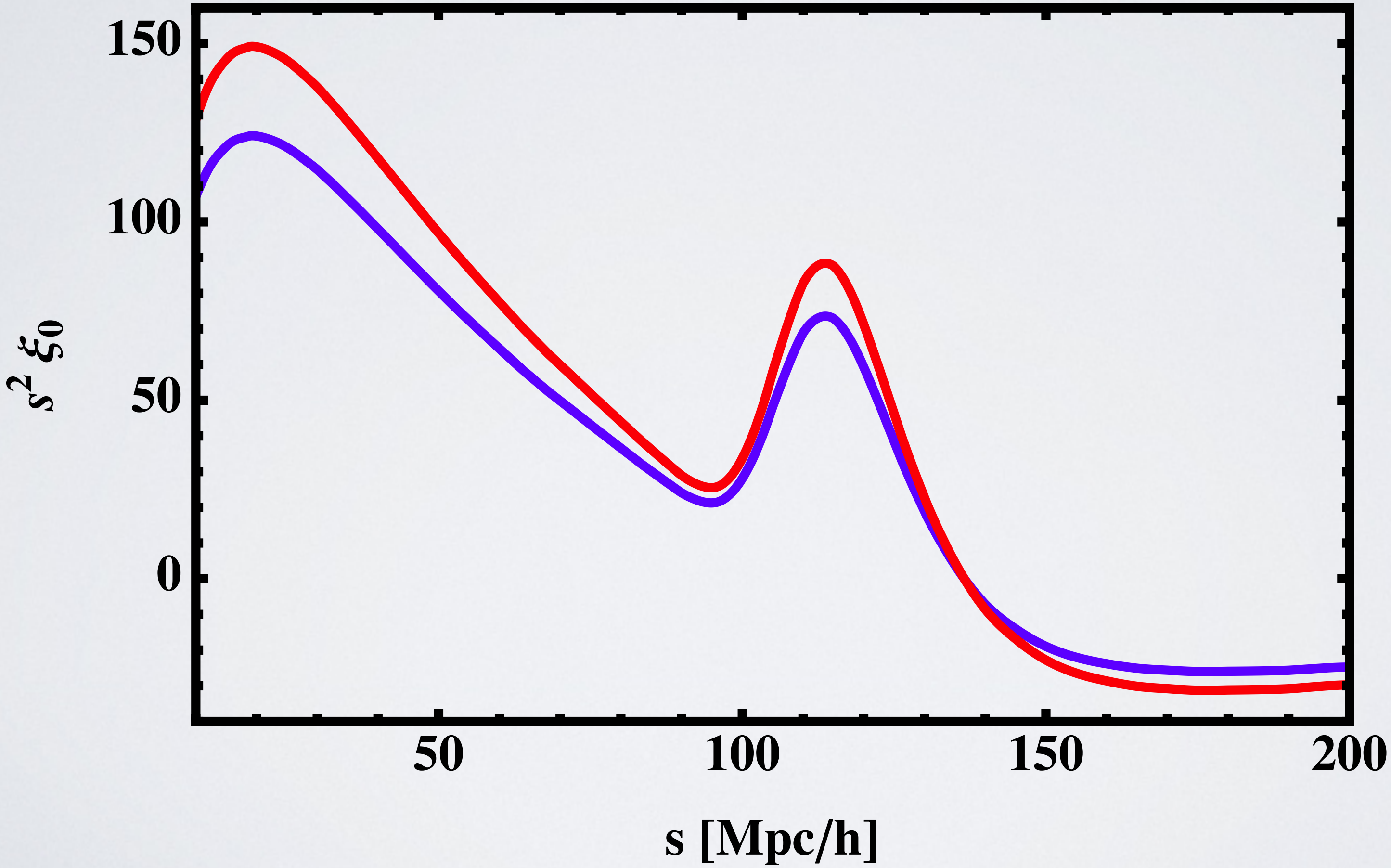
Results: monopole

without redshift distortions

$$\xi_0 = D_1^2 \; b^2 \, \mu_0(s)$$

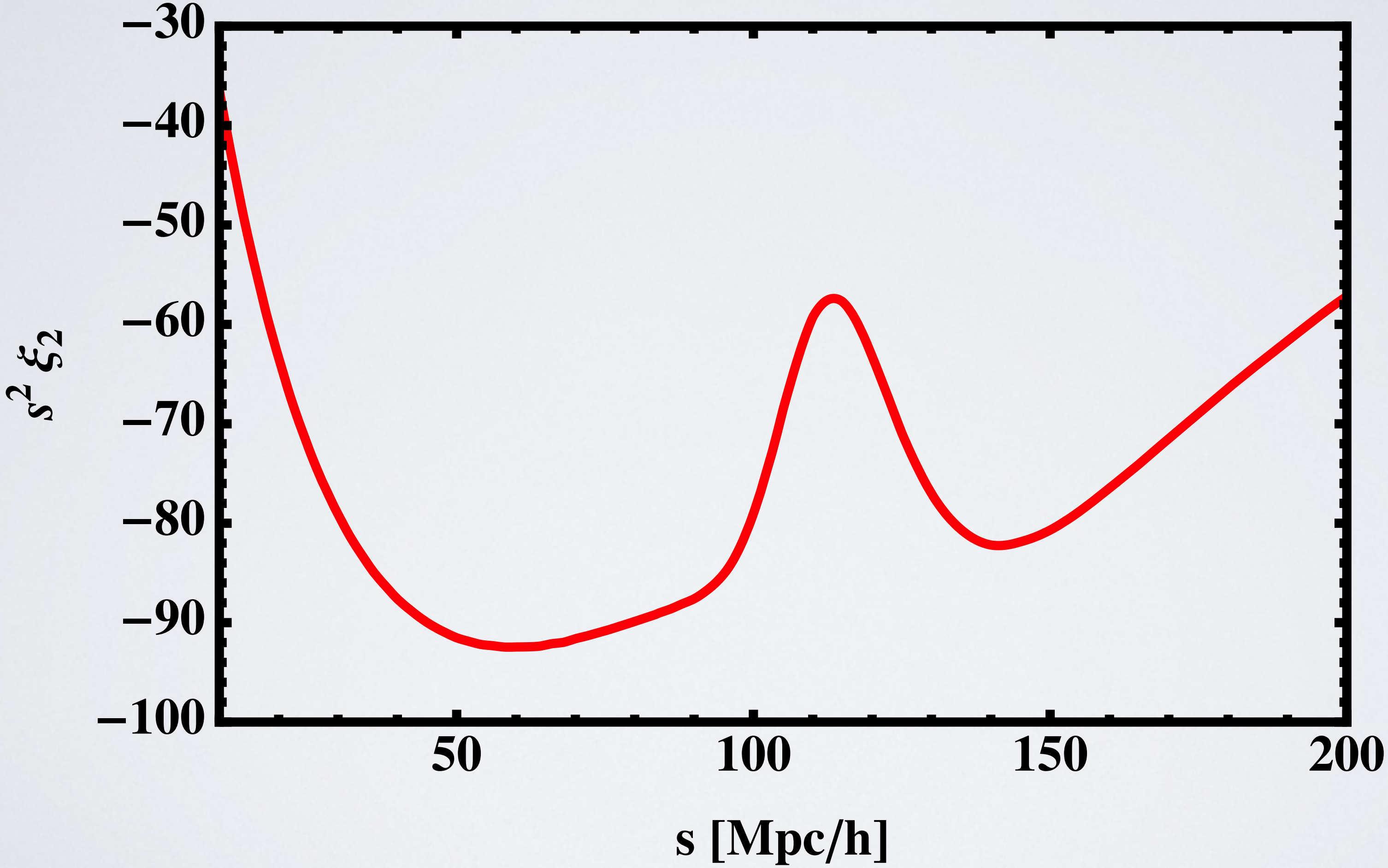
with redshift distortions

$$\xi_0 = D_1^2 \left(b^2 + \frac{2bf}{3} + \frac{f^2}{5} \right) \mu_0(s)$$



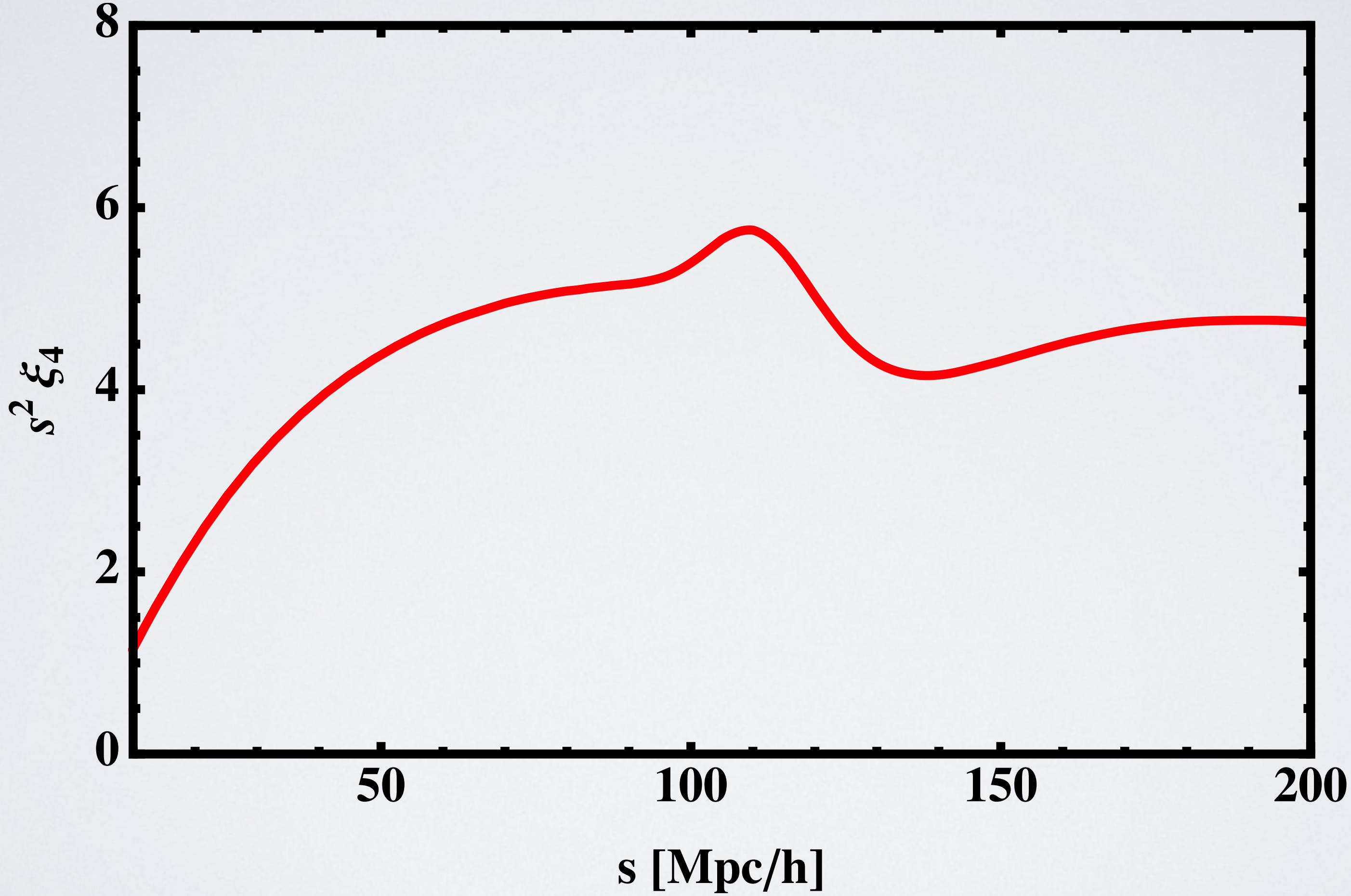
Results: quadrupole

$$\xi_2 = -D_1^2 \left(\frac{4bf}{3} + \frac{4f^2}{7} \right) \mu_2(s)$$



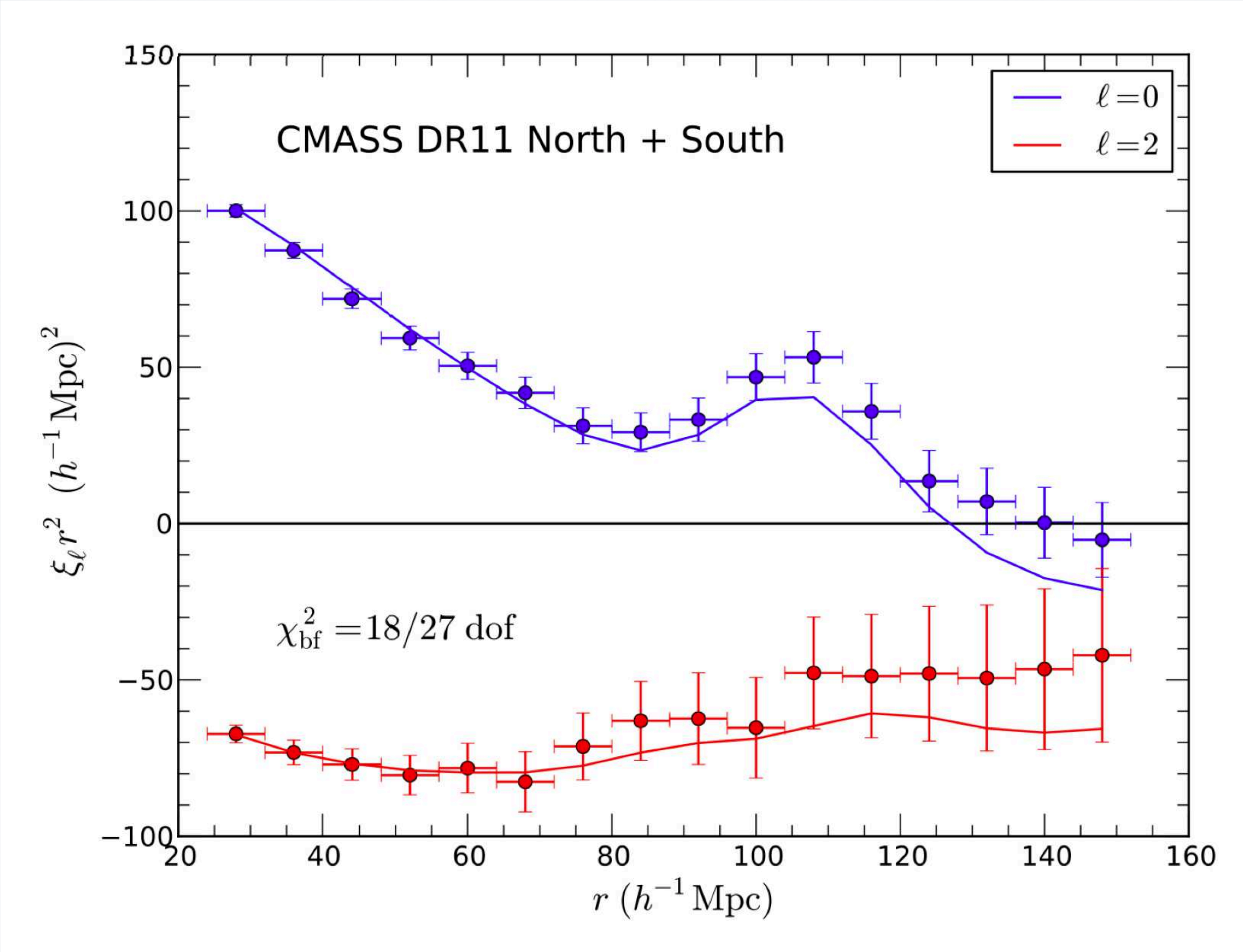
Results: hexadecapole

$$\xi_4 = D_1^2 \frac{8f^2}{35} \mu_4(s)$$



BOSS

L. Samushia et al, arXiv:1312.4899 (2013)



Fourier space

Effect of redshift distortions on the **power spectrum**.

$$\delta_{\rm obs}(\mathbf{k},\eta)=\left(1+(\hat{\mathbf{k}}\cdot\mathbf{n})^2 f\right)\delta(\mathbf{k},\eta)$$

$$P_\delta(k)\,\delta_D(\mathbf{k}+\mathbf{k}')$$

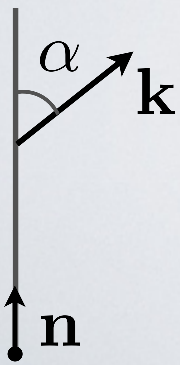
$$\langle\delta_{\rm obs}(\mathbf{k},\eta)\delta_{\rm obs}(\mathbf{k}',\eta)\rangle=\left(1+(\hat{\mathbf{k}}\cdot\mathbf{n})^2 f\right)\left(1+(\hat{\mathbf{k}}'\cdot\mathbf{n}')^2 f\right)\langle\delta(\mathbf{k},\eta)\delta(\mathbf{k}',\eta)\rangle$$

Distant observer approximation: $\mathbf{n}=\mathbf{n}'$

$$P_\delta^{\rm obs}(k,\eta,\cos\alpha)=\left(1+\cos^2(\alpha)\,f^2\right)^2 P_\delta(k,\eta)$$

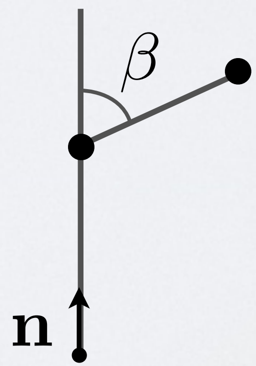
$$\mathbf{n}\cdot\hat{\mathbf{k}}=\cos\alpha$$

Fourier space



Breaking of isotropy: the power spectrum depends on the direction of the Fourier mode.

Real space



Breaking of isotropy: the correlation function depends on the orientation of the pair.

Multipole expansion

- We rewrite the cosine in terms of **Legendre polynomial** (orthogonal basis)

$$P_\delta^{\rm obs}(k,\eta,\cos\alpha)=\left\{1+\frac{2f}{3}+\frac{f^2}{5}+\left(\frac{4f}{3}+\frac{4f^2}{7}\right)P_2(\cos\alpha)\right.$$
$$\left.+\frac{8f^2}{35}P_4(\cos\alpha)\right\}P_\delta(k,\eta)$$

- The density power spectrum **factorises out**.

- In the correlation function this was not the case: different dependence in the separation due to the spherical Bessel functions.

Multipole extraction

- monopole

$$P_\delta^{\mathrm{obs}\,0}(k,\eta) = \frac{1}{2}\int_{-1}^{1} d\mu\; P_\delta^{\mathrm{obs}}(k,\eta,\mu) = \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right) P_\delta(k,\eta)$$

- quadrupole

$$P_\delta^{\mathrm{obs}\,2}(k,\eta) = \frac{5}{2}\int_{-1}^{1} d\mu\, P_2(\mu)\, P_\delta^{\mathrm{obs}}(k,\eta,\mu) = \left(\frac{4f}{3} + \frac{4f^2}{7}\right) P_\delta(k,\eta)$$

- hexadecapole

$$P_\delta^{\mathrm{obs}\,4}(k,\eta) = \frac{9}{2}\int_{-1}^{1} d\mu\, P_4(\mu)\, P_\delta^{\mathrm{obs}}(k,\eta,\mu) = \frac{8f^2}{35} P_\delta(k,\eta)$$

We can measure $f\sigma_8$

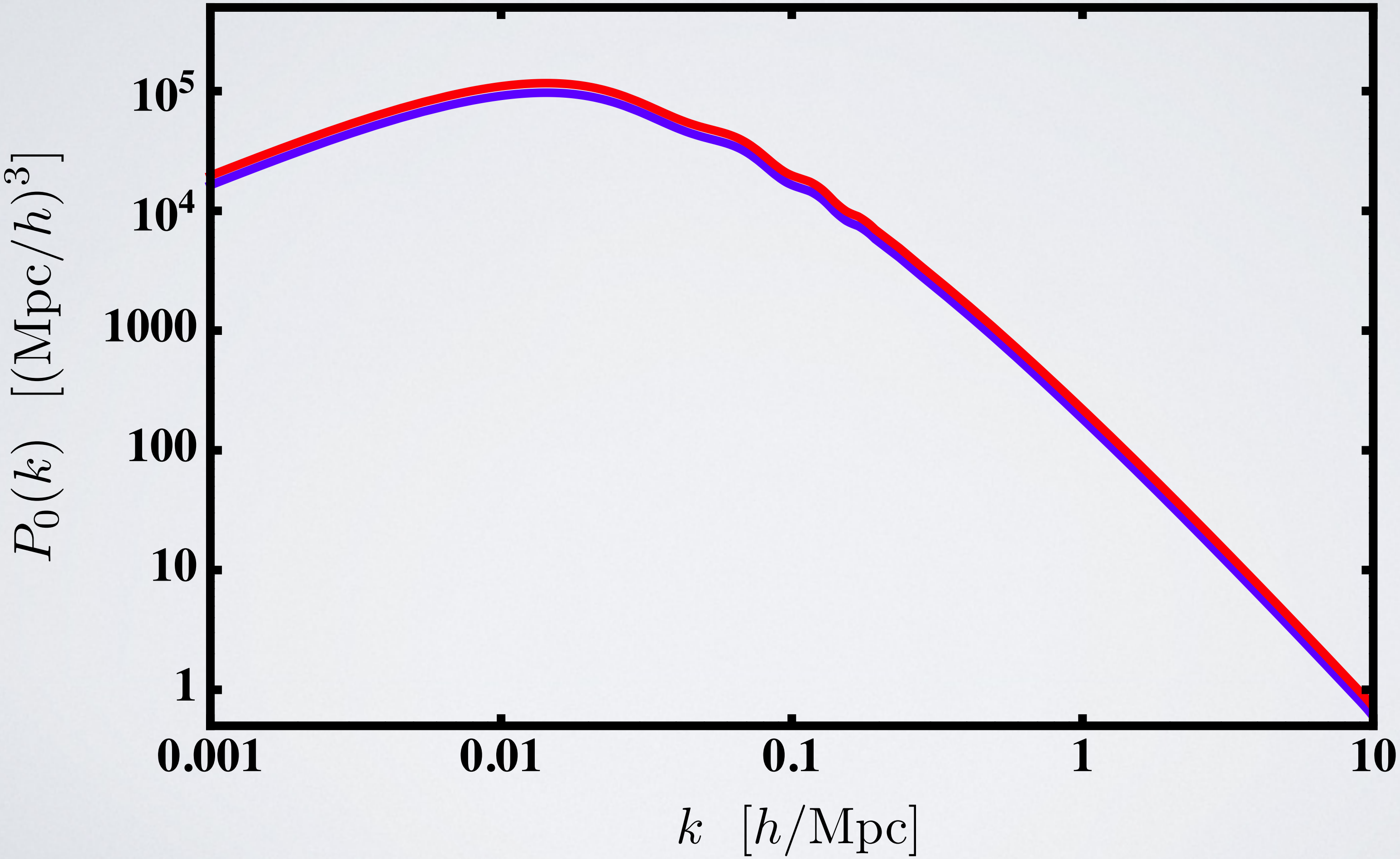
Results: monopole

without redshift distortions

$$b^2 P_\delta(k,\eta)$$

with redshift distortions

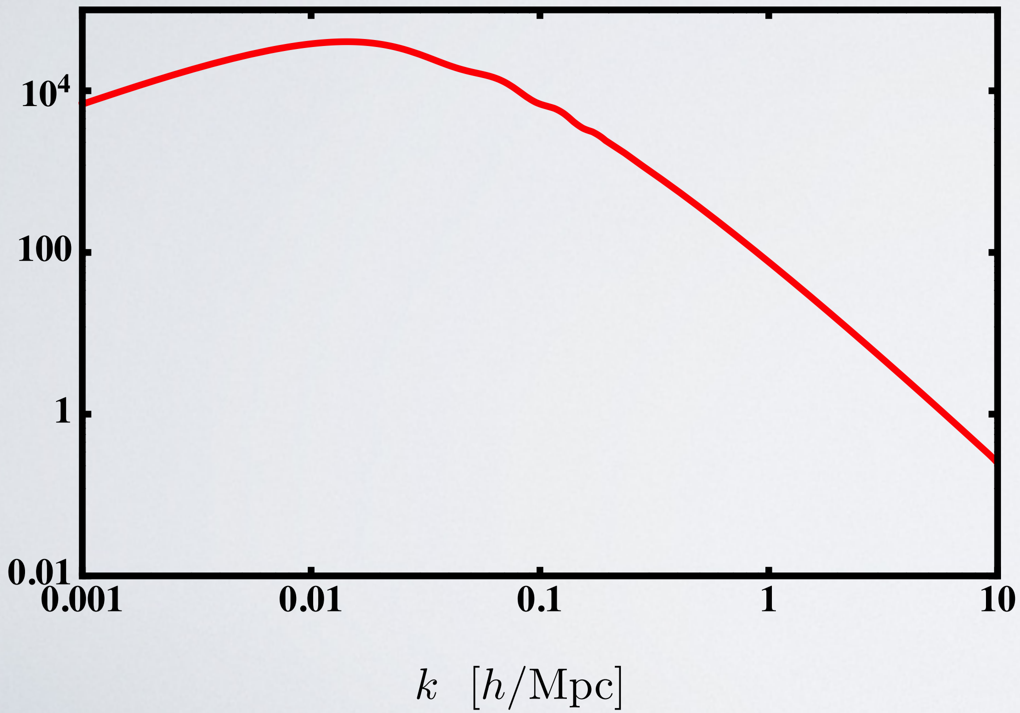
$$\left(b^2 + \frac{2bf}{3} + \frac{f^2}{5}\right) P_\delta(k,\eta)$$



Results: quadrupole and hexadecapole

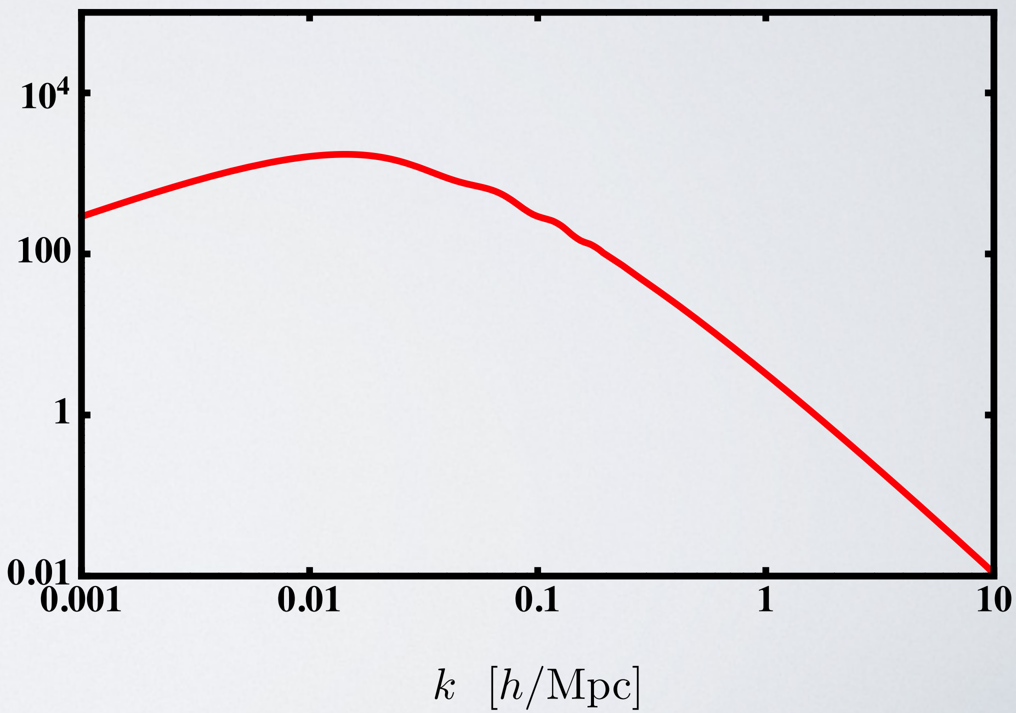
quadrupole

$$\left(\frac{4bf}{3}+\frac{4f^2}{7}\right)P_\delta(k,\eta)$$



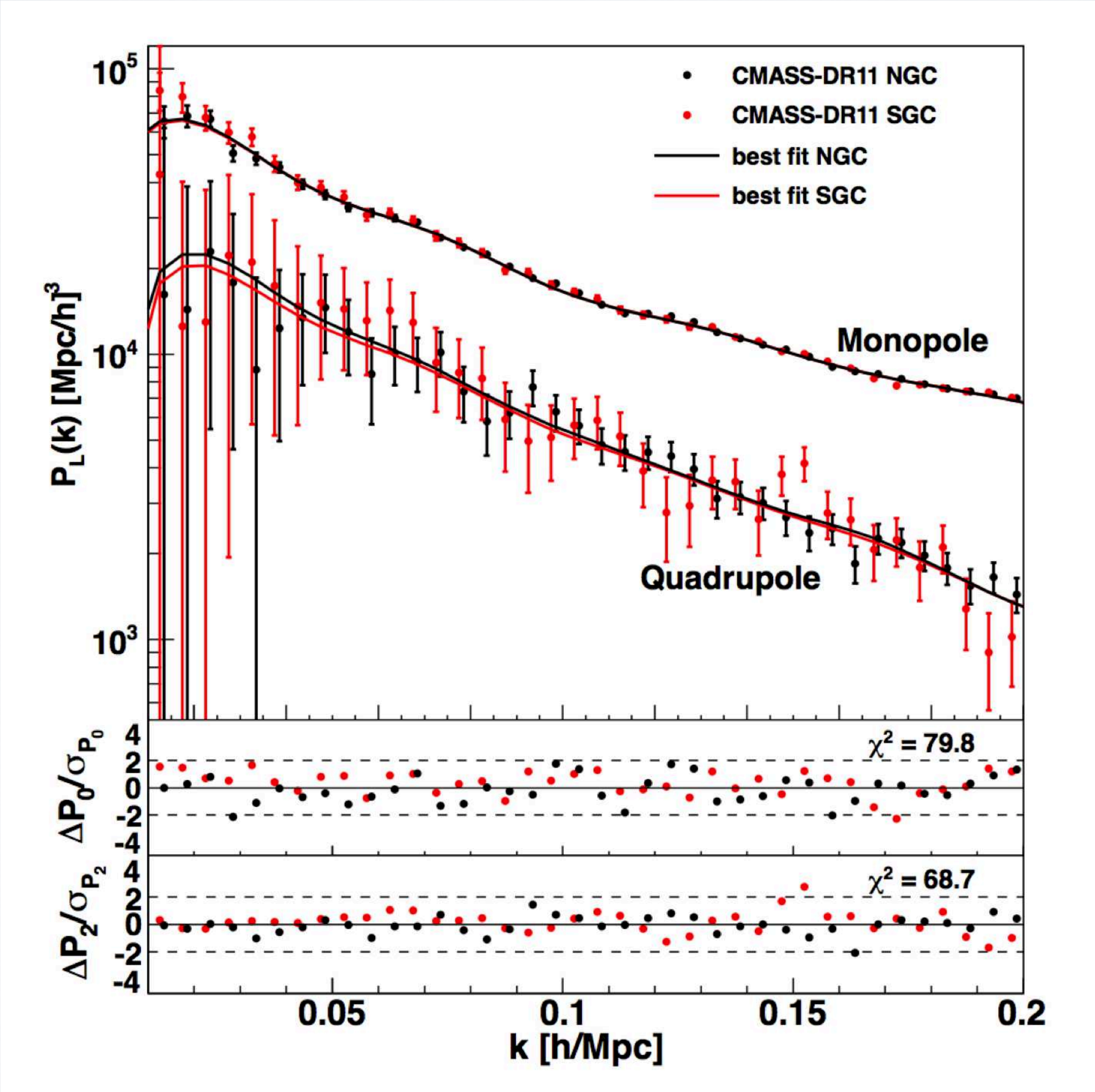
hexadecapole

$$\frac{8f^2}{35}P_\delta(k,\eta)$$



BOSS

F. Beutler et al, arXiv:1312.4611 (2013)



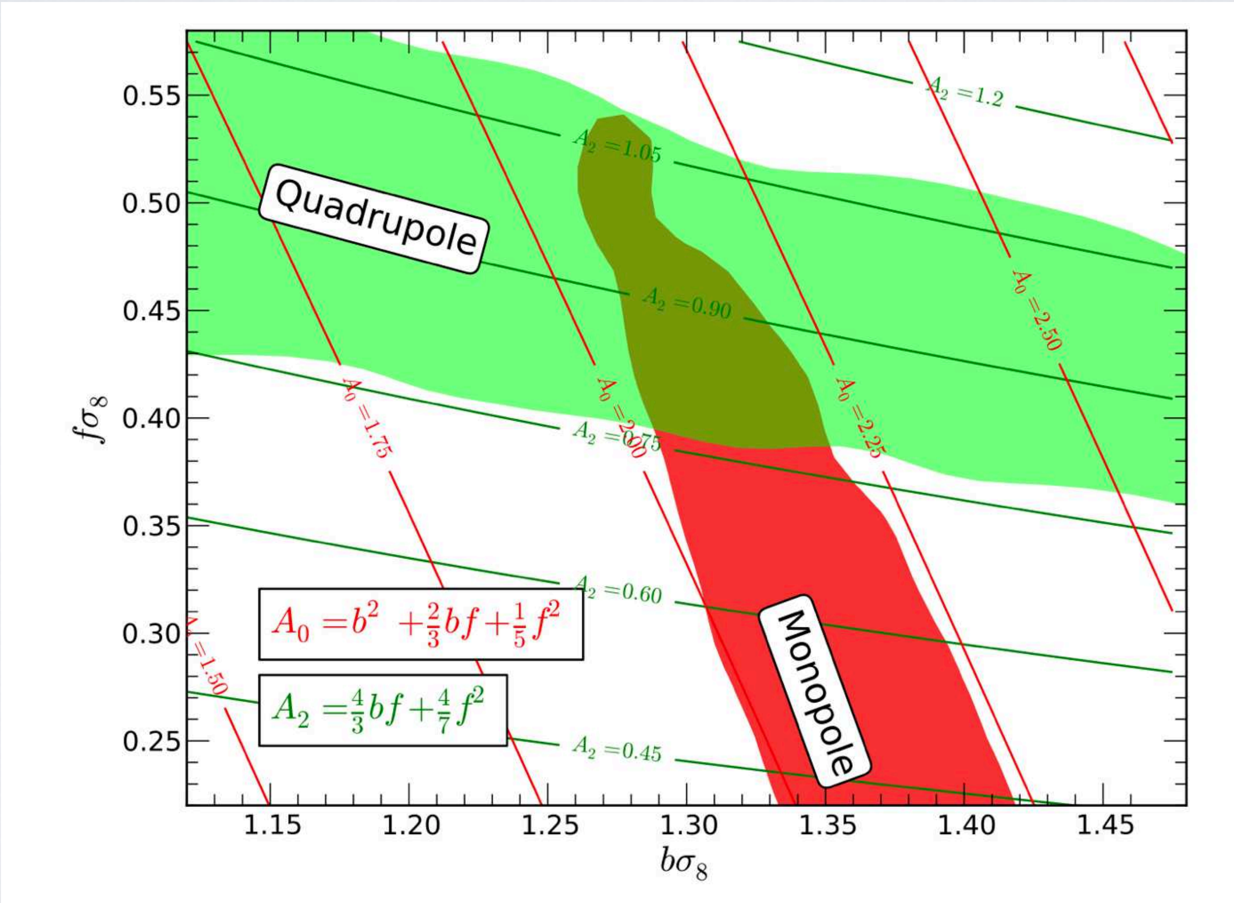
Growth evolution

Which kind of **constraints** can we obtain from redshift distortions?

The monopole and quadrupole allow to measure $f\sigma_8$ and $b\sigma_8$

amplitude of P

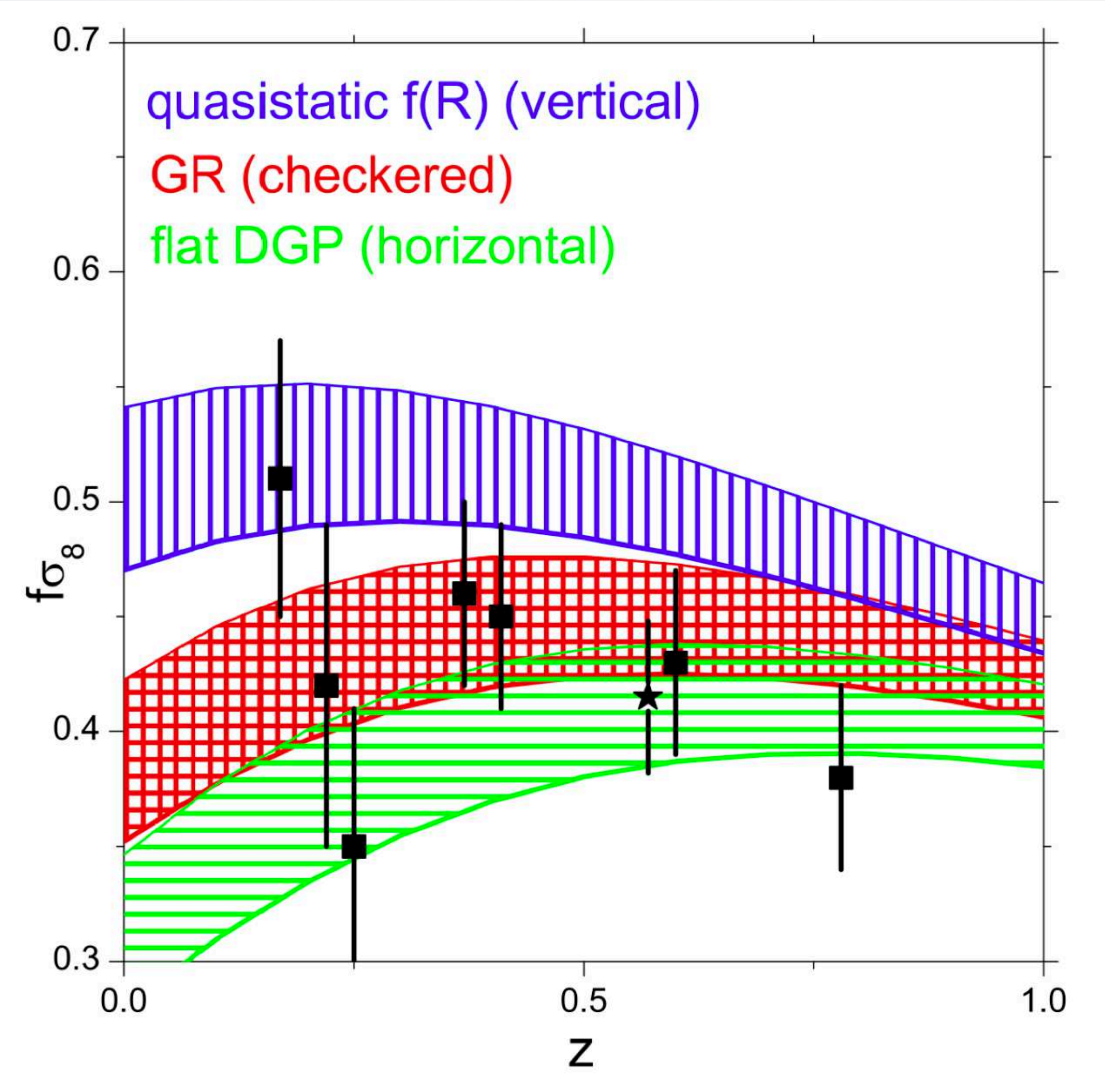
L. Samushia et al, arXiv:1312.4899 (2013)



Modified gravity

f is efficient to constrain **modified** theories of **gravity**: $f = \frac{a}{D_1}\frac{d}{da}D_1$

L. Samushia et al, arXiv:1206.5309 (2012)



No deviations from GR

Consistency of General Relativity

How can we quantify **deviations** from general relativity?

Useful **parameterisation**: $f(a) = \Omega_m(a)^\gamma$

Peebles (1980)
Wang and Steinhardt (1998)

In general relativity with a cosmological constant: $\gamma = 0.55$

Observing a different values would mean a **deviation** from ΛCDM.

This is not a general parameterisation but it allows to test the **consistency** of general relativity.

Consistency of General Relativity

L. Samushia et al, arXiv:1312.4899 (2013)