Full-sky lensing shear at second order

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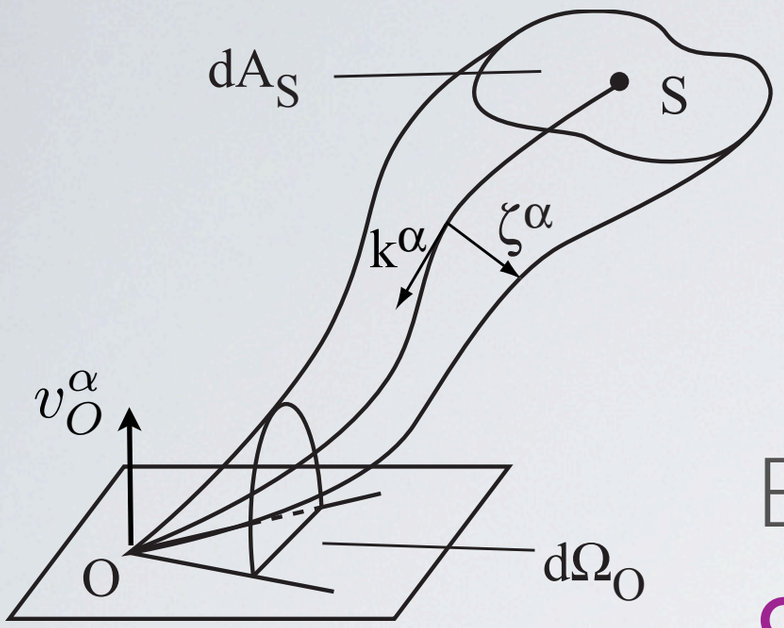
Cosmic shear at large scale

- Weak lensing is a powerful tool to study the distribution of matter. The current surveys are limited to **small patches** in the sky (CFHTLS 170 square degrees).
- Future surveys (Pan-STARRS, DES, LSST, JDEM, Euclid) will cover almost the **all sky** → Measurements of shear correlations at **cosmological scales**.
- At **first order**, the shear is simply related to the transverse variation of the potential, integrated along the trajectory.
- At **second order**, there are various **couplings**:
  - Some couplings dominate at small scale.
  - Other couplings become important when the correlation length is of the order of the depth of the survey.

Outline

- We computed all **second-order** corrections to the shear. Bernardeau, Bonvin and Vernizzi, 2010
- We solved the geodesic deviation equation in a perturbed universe → various **new couplings**
- We are currently computing the **bispectrum** associated with these non-linearities.
- In some configurations, the new couplings are important with respect to the standard couplings.

Sachs equation



k^α photon direction

ξ^α connection vector

$$\frac{D^2\xi^\alpha(\lambda)}{D\lambda^2} = R^\alpha{}_{\beta\mu\nu}k^\beta k^\mu \xi^\nu$$

Sachs 1961

Evolution equation describing the **distortion of the beam**.

We project the equation on a basis $\{v_O^\alpha, k^\alpha, n_1^\alpha, n_2^\alpha\}$

$$\frac{d^2\xi^a}{d\lambda^2} = \mathcal{R}_{ab}\,\xi^b \quad \text{with} \quad \mathcal{R}_{ab} = R_{\alpha\beta\mu\nu}k^\beta k^\mu n_a^\alpha n_b^\nu$$

Solution: $\xi_S^a = \mathcal{D}_{ab}\theta_O^b \qquad \theta_O^b = \xi_O^{b'}$

$\mathcal{D}_{ab}$ **magnification matrix**

Magnification matrix

$$\frac{d^2}{d\lambda^2}\mathcal{D}_{ab} = \mathcal{R}_{ac}\mathcal{D}_{cb} \quad \text{with} \quad \mathcal{R}_{ab} = R_{\alpha\beta\mu\nu}k^\beta k^\mu n_a^\alpha n_b^\nu$$

Convergence $$\kappa = -\frac{1}{2\lambda_S}\left(\mathcal{D}_{11} + \mathcal{D}_{22}\right)$$

Shear $$\gamma = -\frac{1}{2\lambda_S}\left[\mathcal{D}_{11} - \mathcal{D}_{22} + i(\mathcal{D}_{12} + \mathcal{D}_{21})\right]$$

Rotation $$\omega = -\frac{1}{2\lambda_S}\left(\mathcal{D}_{12} - \mathcal{D}_{21}\right)$$

Observable quantity: reduced shear

$$g = \frac{\gamma}{1 - \kappa - i\omega} \simeq \frac{\gamma}{1 - \kappa}$$

First order

$$ds^2 = -(1+2\phi)d\eta^2 + (1-2\psi)dx^2$$

The effect of the expansion $\mathcal{D}_{ab} \to a_S\, \mathcal{D}_{ab}$

$$\gamma(\chi_S) = \int_0^{\chi_S} d\chi\, \frac{\chi_S - \chi}{\chi\chi_S}\, \eth^2 \Psi$$

$$\kappa(\chi_S) \quad = \quad \psi(\chi_S) + \int_0^{\chi_S} d\chi \left(-\frac{2}{\chi_S}\Psi + \frac{\chi_S - \chi}{\chi\chi_S}\, \eth\, \bar{\eth}\Psi \right)$$

First order

$$ds^2 = -(1+2\phi)d\eta^2 + (1-2\psi)dx^2$$

$$\Psi = \frac{1}{2}(\psi+\phi) \qquad \chi = \eta_O - \eta$$

$$\eth = -\sin^s\theta\left[\partial_\theta + \frac{i}{\tan\theta}\partial_\varphi\right](\sin^{-s}\theta)$$

$$\gamma(\chi_S) = \int_0^{\chi_S} d\chi\, \frac{\chi_S - \chi}{\chi\chi_S}\,\eth^2\Psi$$

$$\kappa(\chi_S) \quad = \quad \psi(\chi_S) + \int_0^{\chi_S} d\chi\left(-\frac{2}{\chi_S}\Psi + \frac{\chi_S-\chi}{\chi\chi_S}\,\eth\,\bar{\eth}\Psi\right)$$

First order

$$ds^2 = -(1+2\phi)d\eta^2 + (1-2\psi)dx^2$$

$$\Psi = \frac{1}{2}(\psi+\phi) \qquad \chi = \eta_O - \eta$$

$$\not{\partial} = -\sin^s\theta\left[\partial_\theta + \frac{i}{\tan\theta}\partial_\varphi\right](\sin^{-s}\theta)$$

$\not{\partial}\Psi$	Spin one
$\not{\partial}^2\Psi$	Spin two

$$\gamma(\chi_S) = \int_0^{\chi_S} d\chi\, \frac{\chi_S - \chi}{\chi\chi_S}\, \not{\partial}^2\Psi$$

$$\kappa(\chi_S) \quad = \quad \psi(\chi_S) + \int_0^{\chi_S} d\chi \left(-\frac{2}{\chi_S}\Psi + \frac{\chi_S - \chi}{\chi\chi_S}\, \not{\partial}\,\bar{\not{\partial}}\Psi\right)$$

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$$\kappa(\chi_S) \quad = \quad \psi(\chi_S) + \int_0^{\chi_S} d\chi \left(-\frac{2}{\chi_S}\Psi + \frac{\chi_S - \chi}{\chi\chi_S}\, \eth\, \overset{\Delta_\perp}{\bar{\eth}}\Psi\right)$$

First order

$$ds^2 = -(1+2\phi)d\eta^2 + (1-2\psi)dx^2$$

$$\Psi = \frac{1}{2}(\psi+\phi) \qquad \chi = \eta_O - \eta$$

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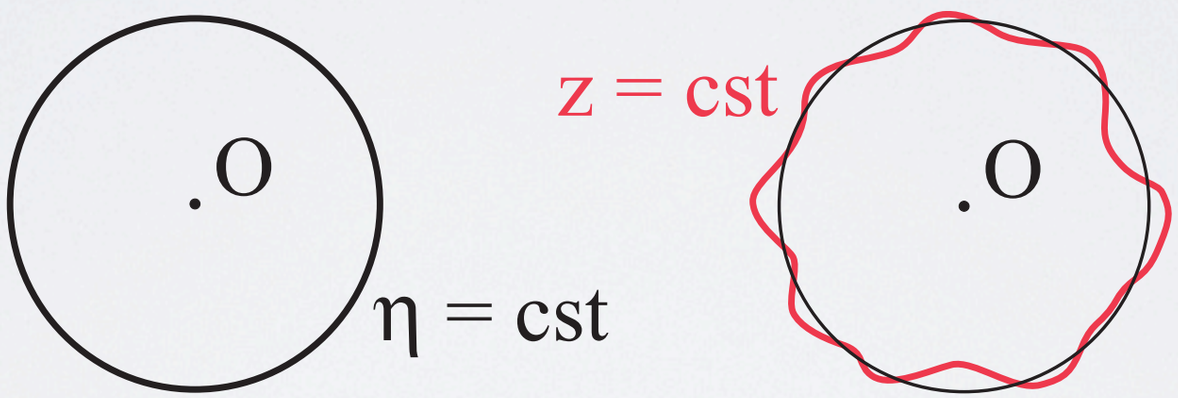
$\Delta_\perp$

Redshift perturbations

The **conformal time** η_S is not an observable quantity.
We need to express $\mathcal{D}_{ab}$ as a function of the **redshift** z_S

In a perturbed Universe, the redshift is perturbed.



- At first order, only the **convergence** is affected.
- At second order, the **shear** is also affected.

Second order

$$\frac{d^2}{d\lambda^2}\mathcal{D}_{ab} = \mathcal{R}_{ac}\mathcal{D}_{cb} \qquad \mathcal{R}_{ab} = R_{\alpha\beta\mu\nu}k^\beta k^\mu n_a^\alpha n_b^\nu$$

- We integrate on **perturbed geodesics**
  → beyond Born approximation.

- We take into account **vector** and **tensor** modes

$$ds^2 = -e^{2\phi}d\eta^2 + 2\omega_i d\eta dx^i + \left(e^{-2\psi}\delta_{ij} + h_{ij}\right)dx^i dx^j$$

- We compute the **reduced shear**: $g = \dfrac{\gamma}{1-\kappa}$

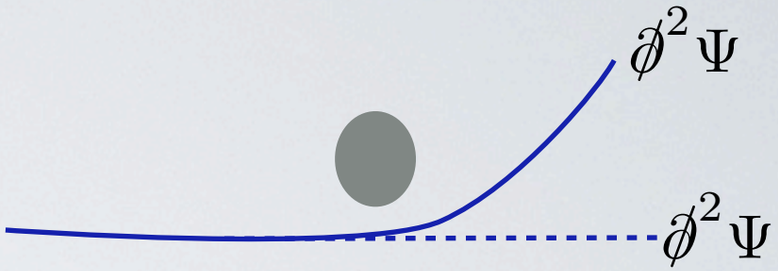
- We take into account the perturbations of the **redshift**.

Results: standard corrections

- Corrections to Born approximation

$$\int_0^{\chi_S} d\chi \frac{1}{\chi\chi_S} \not{\partial}^3\Psi \int_0^{\chi} d\chi' \frac{\chi-\chi'}{\chi'} \bar{\not{\partial}}\Psi$$

$$\ell^3 \quad \times \quad \ell \quad = \quad \ell^4$$



Bernardeau et al. 1997
Cooray and Hu 2002
Dodelson et al. 2005
Shapiro and Cooray 2006

- Lens-lens couplings

$$\int_0^{\chi_S} d\chi \frac{1}{\chi\chi_S} \not{\partial}^2\Psi \int_0^{\chi} d\chi' \frac{\chi-\chi'}{\chi'} \not{\partial}\,\bar{\not{\partial}}\Psi$$

Shear Convergence

New terms

- Intrinsic contribution

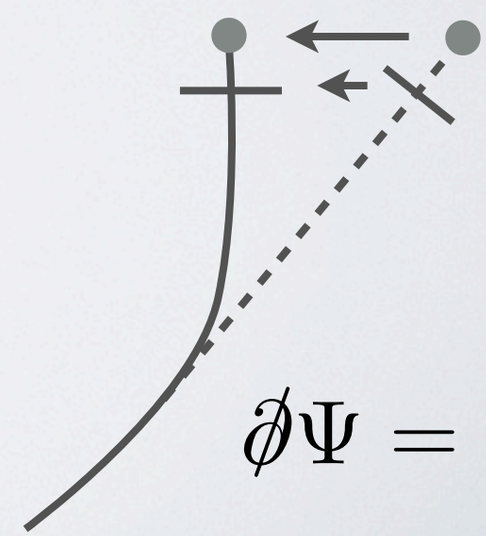
$$\int_0^{\chi_S} d\chi \, \frac{\chi_S - \chi}{\chi \chi_S} \, \not{\partial}^2 \Psi^2(\chi)$$

- Time delay-lens coupling

$$\int_0^{\chi_S} d\chi \, \frac{1}{\chi_S} \Psi(\chi) \int_0^{\chi} d\chi' \, \frac{1}{\chi'} \, \not{\partial}^2 \Psi(\chi')$$

- Deflection-displacement couplings

$$\int_0^{\chi_S} d\chi \, \frac{\chi_S - \chi}{\chi \chi_S} \, \not{\partial} \Psi(\chi) \int_0^{\chi} d\chi' \, \frac{\chi - \chi'}{\chi'} \, \not{\partial} \Psi(\chi')$$



$$\not{\partial} \Psi = c$$

New terms

- Intrinsic contribution

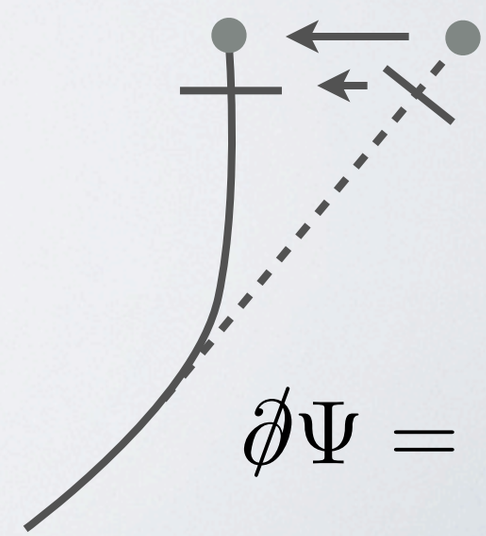
$$\int_0^{\chi_S} d\chi \, \frac{\chi_S - \chi}{\chi \chi_S} \, \not{\partial}^2 \Psi^2(\chi)$$

- Time delay-lens coupling

$$\int_0^{\chi_S} d\chi \, \frac{\chi_S - \chi}{\chi \chi_S} \dot{\Psi}(\chi) \int_0^{\chi} d\chi' \; \not{\partial}^2 \Psi(\chi')$$

- Deflection-displacement couplings

$$\int_0^{\chi_S} d\chi \, \frac{\chi_S - \chi}{\chi \chi_S} \, \not{\partial} \Psi(\chi) \int_0^{\chi} d\chi' \, \frac{\chi - \chi'}{\chi'} \, \not{\partial} \Psi(\chi')$$



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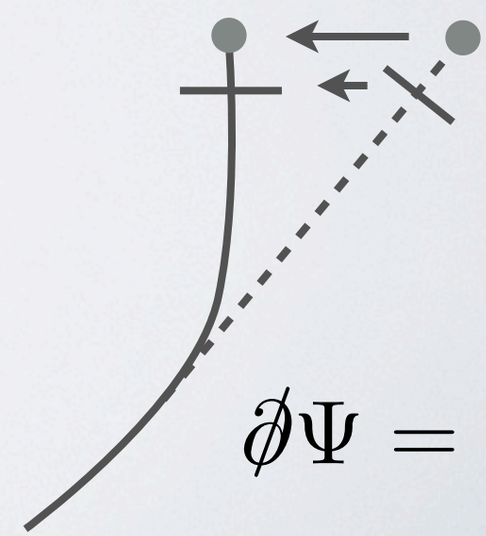
$$\int_0^{\chi_S} d\chi \, \frac{1}{\chi \chi_S} \, \not{\partial}^2 \Psi(\chi) \int_0^{\chi} d\chi' \, \Psi(\chi')$$

- Deflection-displacement couplings

$$\int_0^{\chi_S} d\chi \, \frac{\chi_S - \chi}{\chi \chi_S} \, \not{\partial} \Psi(\chi) \int_0^{\chi} d\chi' \, \frac{\chi - \chi'}{\chi'} \, \not{\partial} \Psi(\chi')$$



$$\not{\partial} \Psi = c$$

New terms

- Reduced shear

$$\int_0^{\chi_S} d\chi \frac{\chi_S - \chi}{\chi \chi_S} \eth \bar{\eth} \Psi \int_0^{\chi_S} d\chi' \frac{\chi_S - \chi'}{\chi' \chi_S} \eth^2 \Psi - 2 \int_0^{\chi_S} d\chi \Psi \int_0^{\chi_S} d\chi' \frac{\chi_S - \chi'}{\chi' \chi_S^2} \eth^2 \Psi$$

- Redshift perturbations

$$\frac{1 + z_S}{\chi_S^2 H_S} \left(\phi(\chi_S) + \mathbf{n} \cdot \mathbf{v}_S - 2 \int_0^{\chi_S} d\chi \, \dot{\Psi} \right) \int_0^{\chi_S} d\chi \, \eth^2 \Psi$$

- Vector and tensor

$$\frac{1}{2} {}_2h(\chi_S) + \int_0^{\chi_S} d\chi \left[\frac{\chi_S - \chi}{\chi \chi_S} \eth^2 (\omega_r + \frac{1}{2} h_{rr}) + \frac{1}{\chi} \eth ({}_1\omega + {}_1h_r) \right]$$

Bispectrum

Bernardeau, Bonvin, Van de Rijt and Vernizzi, in preparation.

$$g(z_S, \mathbf{n}) = \sum_{\ell m} {}_2a_{\ell m}(z_S) \cdot {}_2Y_{\ell m}(\mathbf{n})$$

E-modes $$E(z_S, \mathbf{n}) = -\frac{1}{2} \sum_{\ell m} ({}_2a_{\ell m} + {}_{-2}a_{\ell m}) Y_{\ell m}(\mathbf{n})$$

B-modes $$B(z_S, \mathbf{n}) = \frac{\mathrm{i}}{2} \sum_{\ell m} \left({}_2a_{\ell m} - {}_{-2}a_{\ell m}\right) Y_{\ell m}(\mathbf{n})$$

At first order, the shear contains only E-modes.
At second order it contains both.

second order ↗ first order ↗ ↖ + perm. $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$

$$\langle E(z_S, \mathbf{n}_1) E(z_S, \mathbf{n}_2) E(z_S, \mathbf{n}_3) \rangle =$$

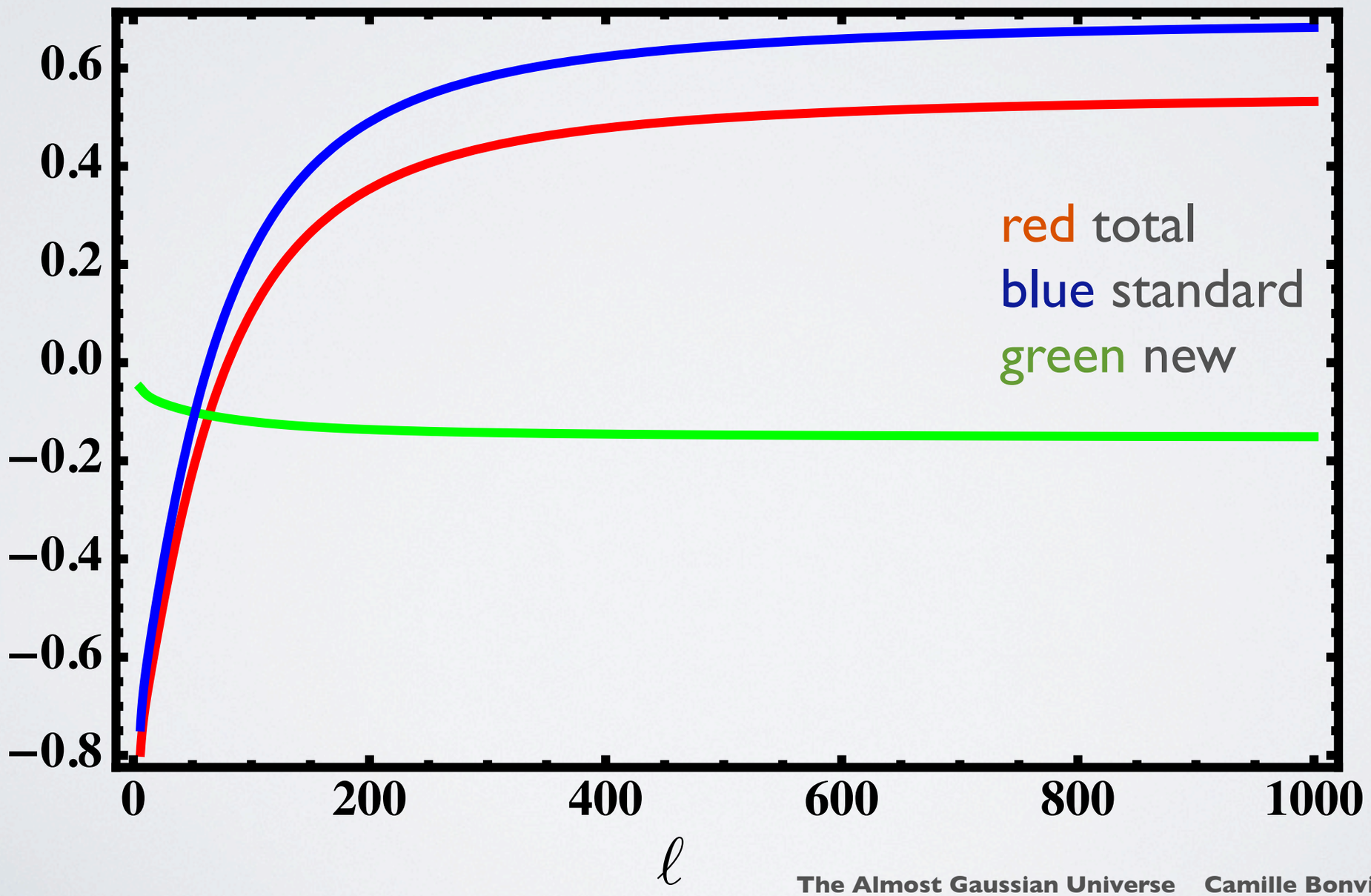
$$\sum_{\substack{\ell_1 \ell_2 \ell_3 \\ m_1 m_2 m_3}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{\ell_1 \ell_2 \ell_3}(z_S) Y_{\ell_1 m_1}(\mathbf{n}_1) Y_{\ell_2 m_2}(\mathbf{n}_2) Y_{\ell_3 m_3}(\mathbf{n}_3)$$

Squeezed configuration

$$\sqrt{\frac{(2\ell_1+1)(2\ell_2+2)(2\ell_3+1)}{4\pi}}\begin{pmatrix}\ell_1 & \ell_2 & \ell_3\\ 0 & 0 & 0\end{pmatrix}$$

$$\frac{b_{4\ell\ell}}{C_\ell C_\ell + 2C_\ell C_4}$$

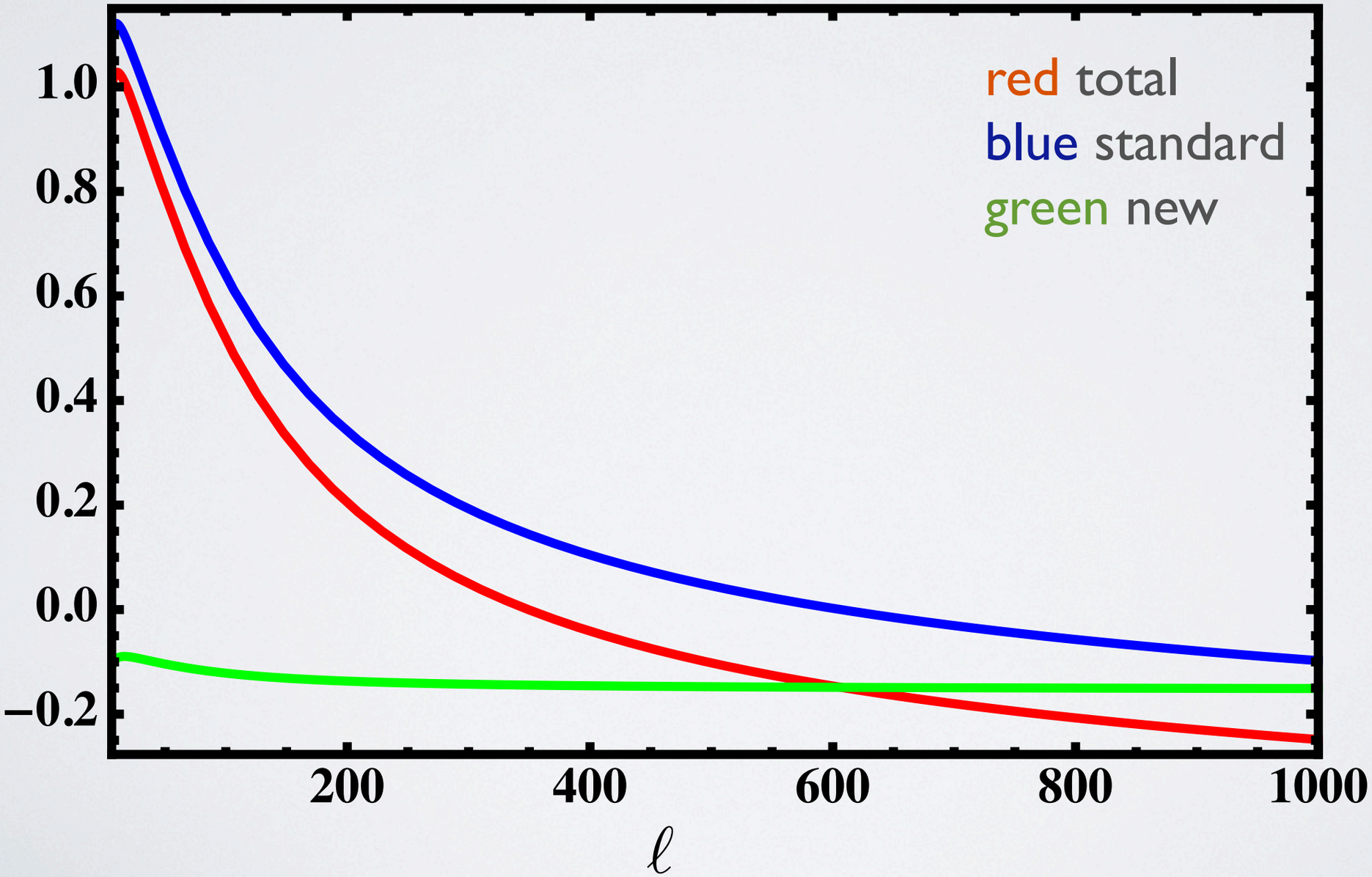
Standard corrections:
- Born corrections
- Lens-lens couplings
- Reduced shear



Squeezed, flat configuration

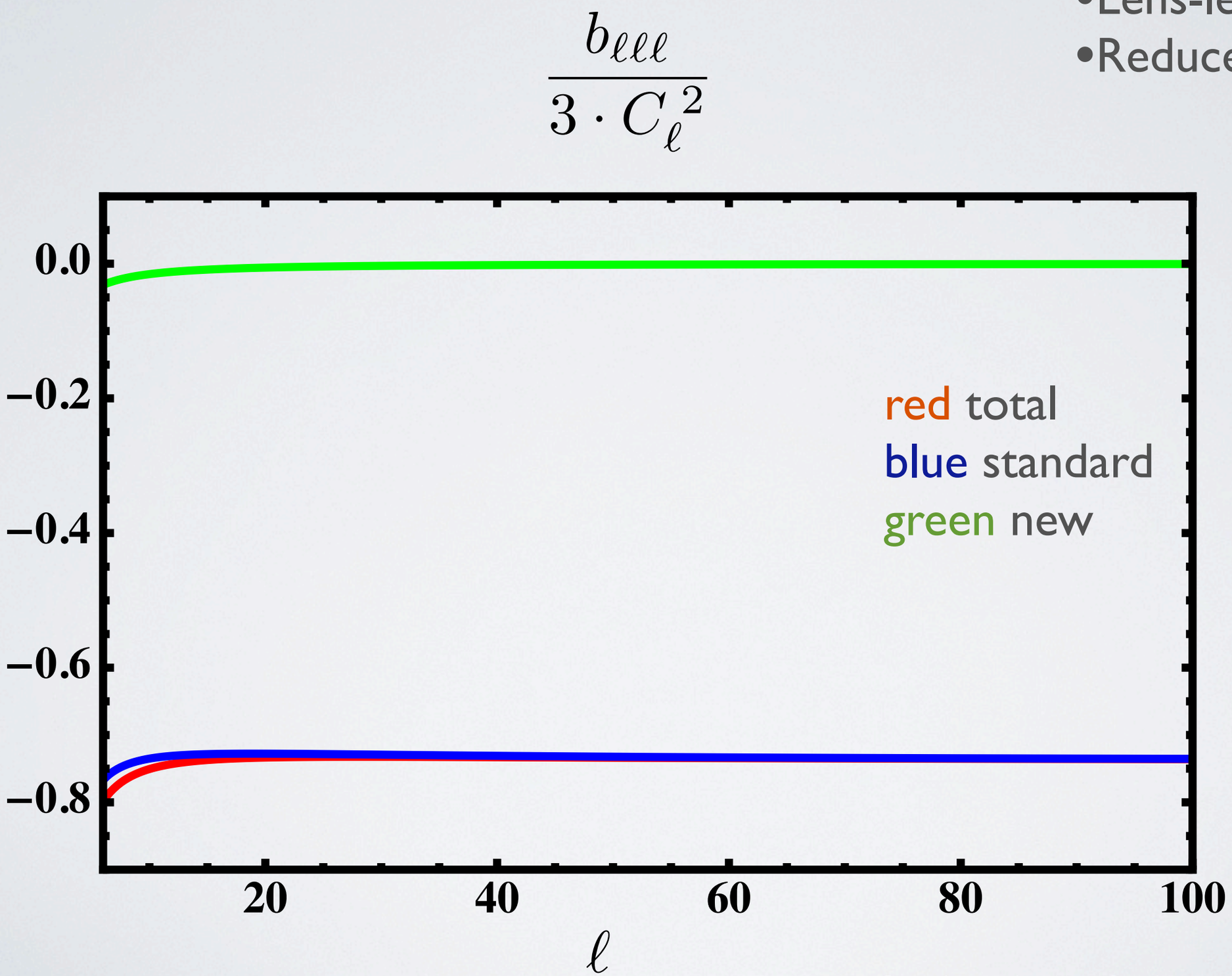
Standard corrections:
- Born corrections
- Lens-lens couplings
- Reduced shear

$$\frac{b_{4\ell\,\ell+4}}{C_\ell C_{\ell+4} + C_\ell C_4 + C_{\ell+4} C_4}$$



Equilateral configuration

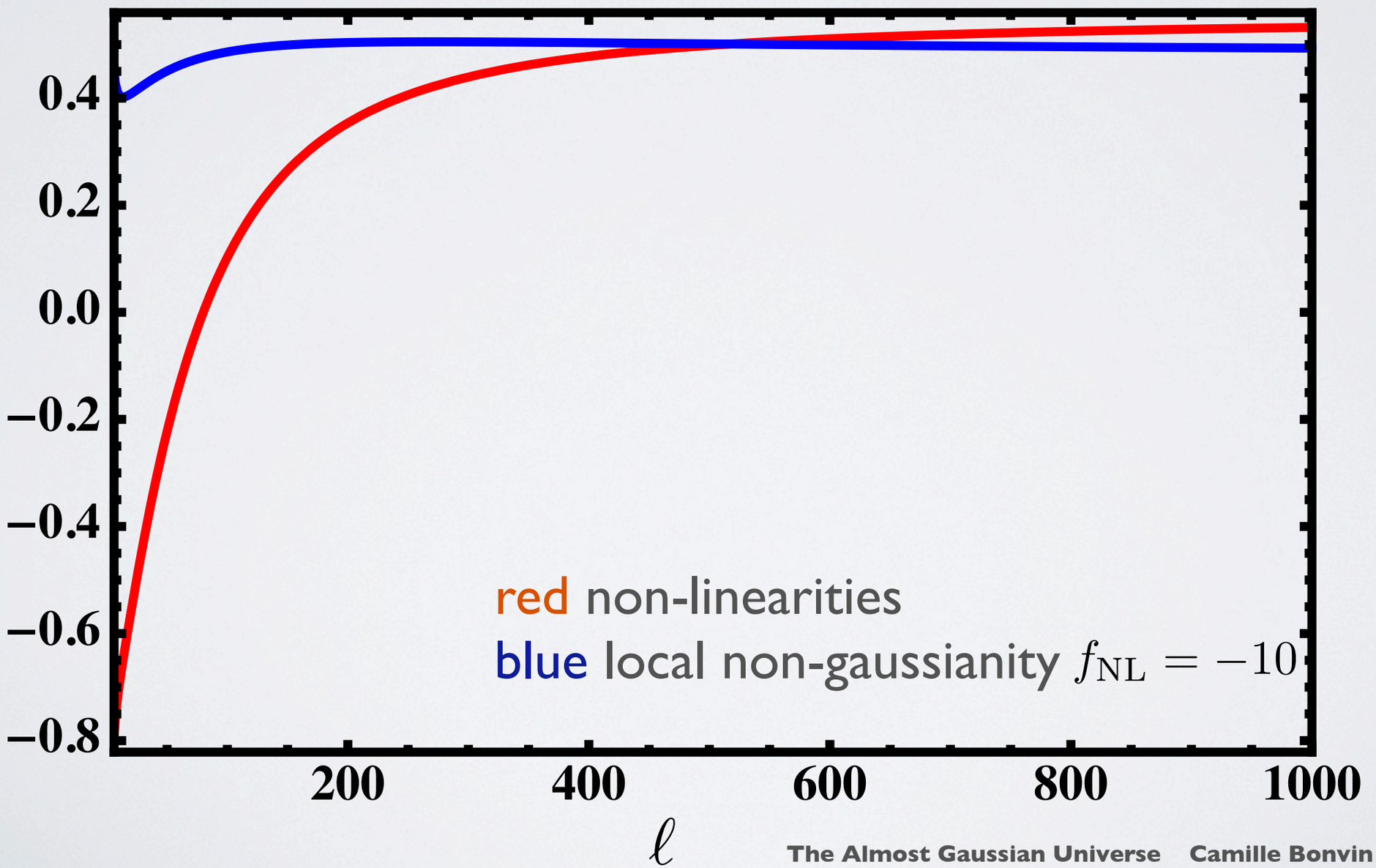
Standard corrections:
- Born corrections
- Lens-lens couplings
- Reduced shear



Local primordial non-gaussianities

$$\Psi = \Psi_{\rm L} + f_{\rm NL}\left(\Psi_{\rm L}^2 - \langle \Psi_{\rm L}^2 \rangle\right) \quad \rightarrow \quad \gamma = \int_0^{\chi_S} d\chi \, \frac{\chi_S - \chi}{\chi \chi_S} \, \phi^2 \Psi$$

$$\frac{b_{4\ell\ell}}{C_\ell C_\ell + 2 C_\ell C_4}$$



Conclusion

- We found various second-order couplings in the shear, that can be split in **two classes**:

- The couplings with 4 transverse derivatives, that dominate at **small scales**. They completely determine the signal in the equilateral configuration.

- The couplings with 2 transverse derivatives, that become important at **large scales**:
  - in the squeezed limit, they contribute to 20 percents.
  - in the flat squeezed limit, they dominate.

- In the **future**, we want to understand the **scaling**. We also need to compute the **dynamical** second-order scalar, vector and tensor part, and to determine the dependence of the bispectrum on **cosmological parameters**.

Primordial non-gaussianities: equilateral

$$\frac{b_{\ell\ell\ell}}{3 \cdot C_\ell^2}$$