

Effect of weak lensing on GWs

Camille Bonvin

Institute of Theoretical Physics
CEA-Saclay, France

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Outline

- ◆ Effect of **large-scale structure** on GWs emitted by a binary system.
 - Impact on the redshift → impact on the **frequency** and on the chirp **mass**.
 - Impact on the **luminosity distance**.
- ◆ At small redshift, peculiar **velocities** dominate.
- ◆ At large redshift, the **lensing** term dominates.

Binary system in an euclidien universe

$$h_+(t) = \frac{2}{r} M_c^{5/3} \left(\pi f(t_{\text{ret}}) \right)^{2/3} (1 + \cos^2 \varphi) \cdot \cos \Phi(t_{\text{ret}})$$

$$h_\times(t) = \frac{4}{r} M_c^{5/3} \left(\pi f(t_{\text{ret}}) \right)^{2/3} \cos \varphi \cdot \sin \Phi(t_{\text{ret}}) \quad t_{\text{ret}} = t - r$$

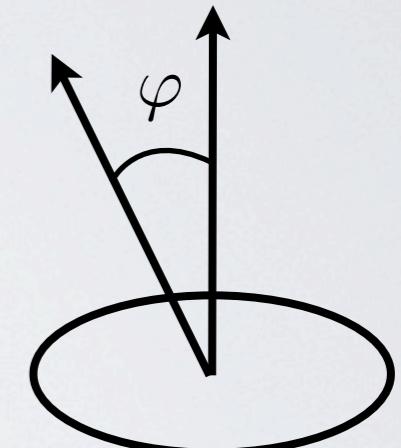
Chirp mass

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Wave form

$$\Phi(t_{\text{ret}}) = 2\pi \int^{t_{\text{ret}}} dt f(t)$$

$$\Phi(\tau) = -2(5M_c)^{-5/8} \tau^{5/8} + \phi_0 \quad \tau = t_{\text{coal}} - t$$



In a non-euclidien universe, both the **frequency** and the **trajectory** of the wave are affected by the geometry.

Time and frequency

- ◆ **Time unit** depends on the reference frame. The redshift is the quantity that relates the time unit in two reference frames

$$dt_S = \frac{dt_O}{1+z}$$

- ◆ The **frequency** of the wave is therefore modified

$$f_S = f_O(1+z)$$

- ◆ The **wave form** is also affected

$$\Phi(\tau_O) = -2 \left(5M_c(1+z) \right)^{-5/8} \tau_O^{5/8} + \phi_0$$

Distance

In an euclidien universe, $1/r$ comes from the wave
propagation equation

$$\square h_+ = \square h_\times = 0 \quad \text{with} \quad \square = \partial_r^2 - \partial_t^2$$

See e.g. Maggiore, Gravitational Waves, 2008

In a non-euclidien universe $\square = g^{\mu\nu} D_\mu D_\nu$

$$\frac{1}{r} \rightarrow \frac{1+z}{d_L}$$

Luminosity distance: $d_L \equiv \sqrt{\frac{\mathcal{L}}{4\pi\mathcal{F}}}$

The **luminosity distance** tells us how the energy emitted by the source is spread during the propagation in a generic universe.

Generic formula

$$h_+ = \frac{2}{d_L} (1+z)^{5/3} M_c^{5/3} \left(\pi f_O(t_{\text{ret}}) \right)^{2/3} (1 + \cos^2 \varphi) \cdot \cos \Phi_O(t_{\text{ret}})$$

$$h_\times = \frac{4}{d_L} (1+z)^{5/3} M_c^{5/3} \left(\pi f_O(t_{\text{ret}}) \right)^{2/3} \cos \varphi \cdot \sin \Phi_O(t_{\text{ret}})$$

Redshifted chirp mass $\mathcal{M}_c = (1+z)M_c$

We compute the impact of **large-scale structure** on gravitational waves at first order in perturbation theory.

$$ds^2 = - (1 + 2\Psi(t, \mathbf{x})) dt^2 + a^2(t) (1 - 2\Psi(t, \mathbf{x})) \delta_{ij} dx^i dx^j$$

$\Psi(t, \mathbf{x})$ gravitational potential

Generic formula

$$h_+ = \frac{2}{d_L} \mathcal{M}_c^{5/3} \left(\pi f_O(t_{\text{ret}}) \right)^{2/3} (1 + \cos^2 \varphi) \cdot \cos \Phi_O(t_{\text{ret}})$$

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Redshift

We solve the null **geodesic equation** $1 + z_S = \frac{E_S}{E_O}$

$$1 + z_S = \frac{a_O}{a_S} \left(1 + \Psi_O - \Psi_S + \mathbf{n} \cdot (\mathbf{v}_O - \mathbf{v}_S) + 2 \int_0^{\chi_S} d\chi \dot{\Psi} \right)$$

- ◆ Gravitational redshift $\Psi_O - \Psi_S$
- ◆ Doppler effect $\mathbf{v}_O - \mathbf{v}_S$
- ◆ Integrated effect $\int_0^{\chi_S} d\chi \dot{\Psi}$ vanishes in a CDM universe.

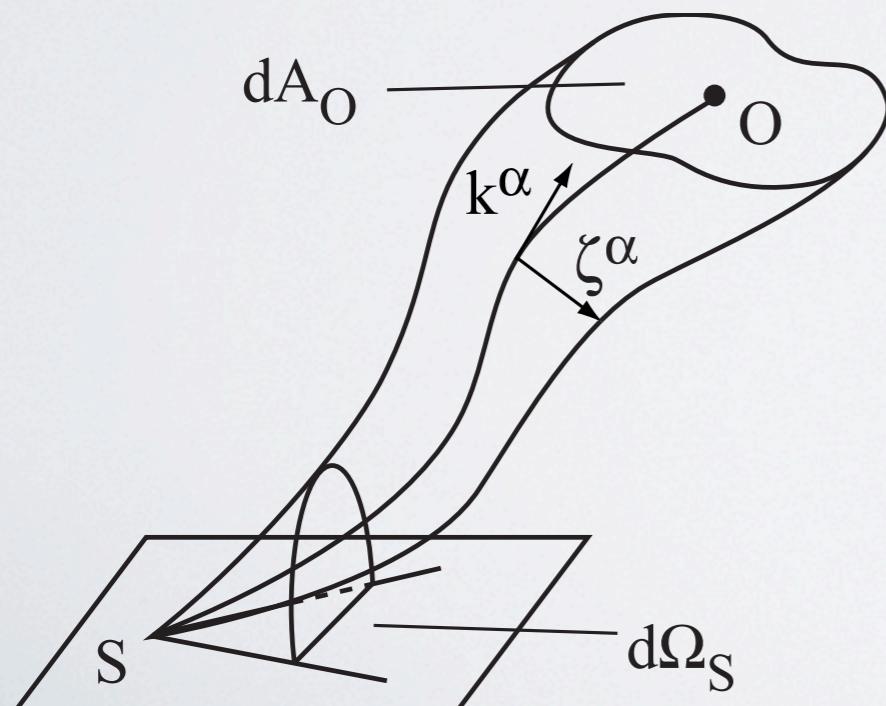
Redshift perturbations affect the **mass** and the **frequency**

Luminosity distance

$$d_L \equiv \sqrt{\frac{\mathcal{L}}{4\pi\mathcal{F}}}$$

\mathcal{L} Luminosity
 \mathcal{F} Flux

We need to know how a beam of GWs is **distorted** by the large-scale structure between the source and the observer.



Homogeneous and isotropic
expanding universe

$$d_L(z_S) = \frac{a_O}{a_S} \int_0^{z_S} \frac{dz}{H(z)}$$

Luminosity distance perturbations

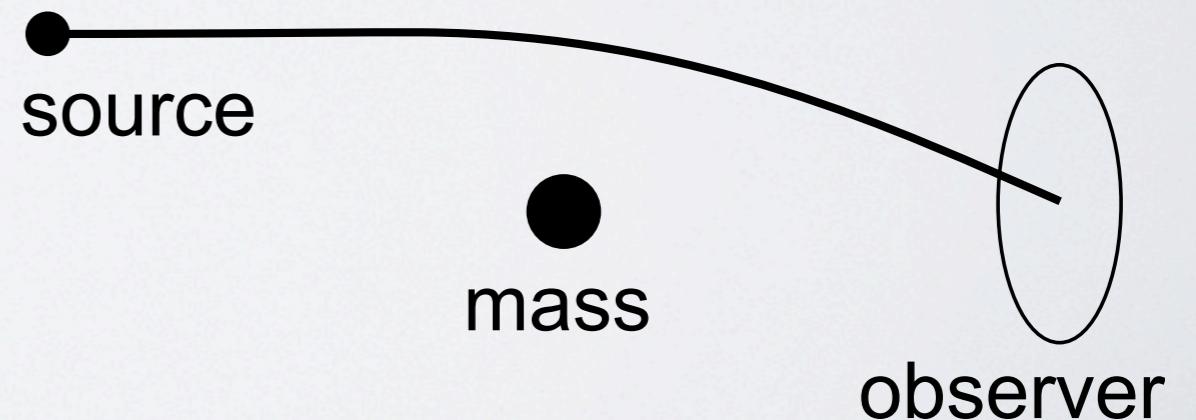
Bonvin, Durrer and Gasparini, 2006

$$\begin{aligned} d_L = & (1 + z_S) \chi_S \left\{ 1 + \frac{1}{\mathcal{H}_S \chi_S} \mathbf{v}_O \cdot \mathbf{n} + \left(1 - \frac{1}{\mathcal{H}_S \chi_S} \right) \mathbf{v}_S \cdot \mathbf{n} \right. \\ & + \left(\frac{1}{\mathcal{H}_S \chi_S} - 2 \right) \Psi_S - \left(\frac{1}{\mathcal{H}_S \chi_S} + 1 \right) \Psi_O \\ & + \frac{4}{\chi_S} \int_0^{\chi_S} d\chi \Psi + \frac{2}{\chi_S} \int_0^{\chi_S} d\chi \left(\frac{1}{\mathcal{H}_S} - \chi \right) \dot{\Psi} \\ & \left. + \int_0^{\chi_S} d\chi \frac{(\chi - \chi_S)\chi}{\chi_S} \Delta_{\perp} \Psi \right\} \end{aligned}$$

Various contributions

- ◆ Local terms $\Psi_O \ \Psi_S$
- ◆ Doppler term $\mathbf{v}_O \ \mathbf{v}_S$
- ◆ Integrated terms along the trajectory $\dot{\Psi} \ \ \Psi$
- ◆ Lensing term

$$\int_0^{\chi_s} d\chi \frac{(\chi - \chi_s)\chi}{\chi_s} \Delta_{\perp} \Psi$$



Angular power spectrum

We compute the **amplitude** and **scale-dependence** of the perturbations.

We assume a statistically homogeneous and isotropic gravitational potential, gaussian and with a flat spectrum.

We compute the **angular power spectrum**.

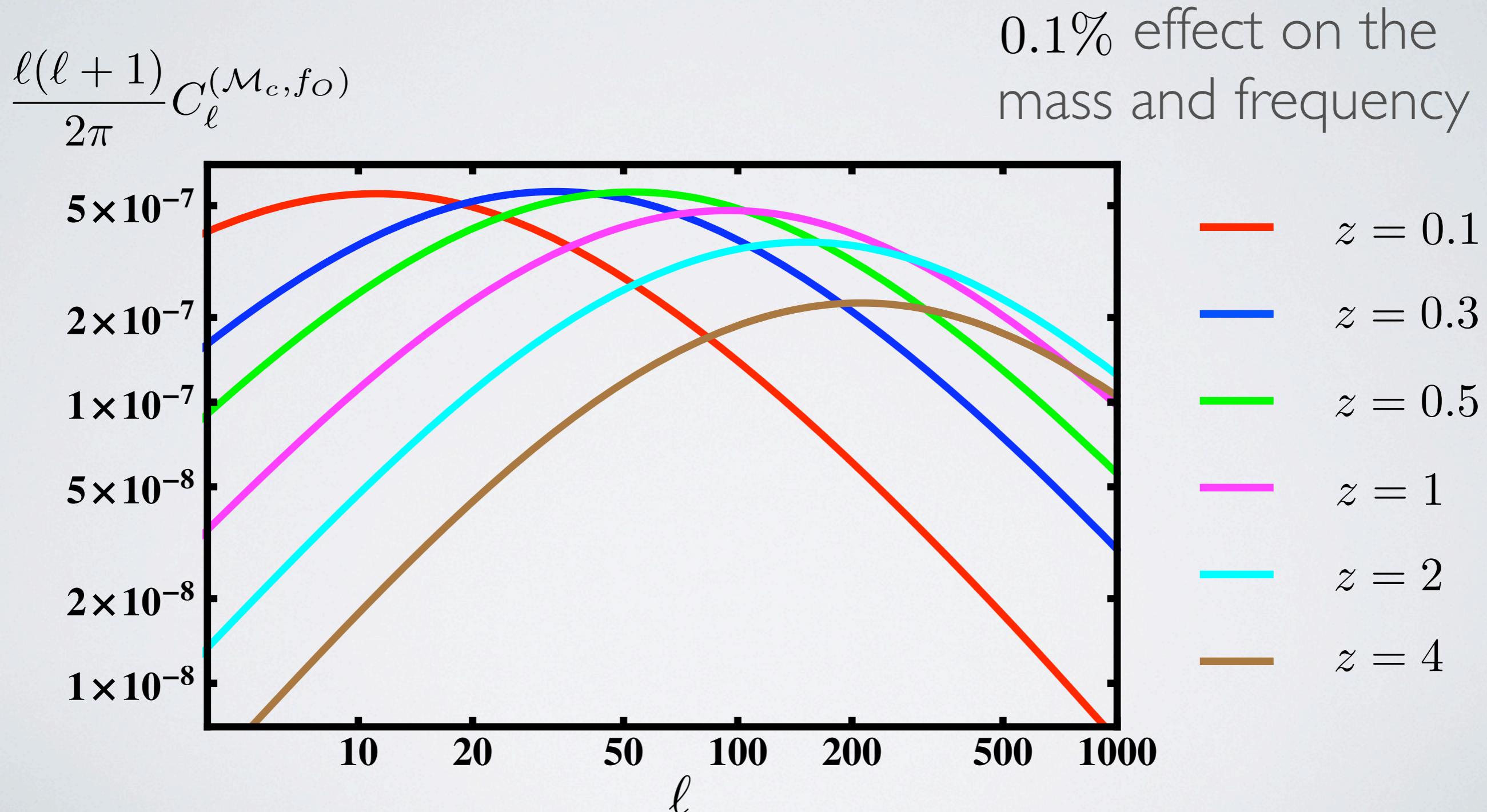
$$F(z_S, \mathbf{n}) = \sum_{\ell m} a_{\ell m}^F(z_S) Y_{\ell m}(\mathbf{n}) \quad F = d_L, \mathcal{M}_c, f_O$$

$$\langle F(z_S, \mathbf{n}) F(z_S, \mathbf{n}') \rangle = \sum_{\ell} \frac{2\ell + 1}{4\pi} C_{\ell}^F(z_S) P_{\ell}(\mathbf{n} \cdot \mathbf{n}')$$

- ◆ At small redshift: peculiar **velocities** dominate.
- ◆ At large redshift: **lensing** dominates.

Peculiar velocities on the redshift

$$\frac{\delta z_S}{1 + z_S} = -\mathbf{v}_S \cdot \mathbf{n} = \frac{\delta \mathcal{M}_c}{\mathcal{M}_c} = -\frac{\delta f_O}{f_O}$$

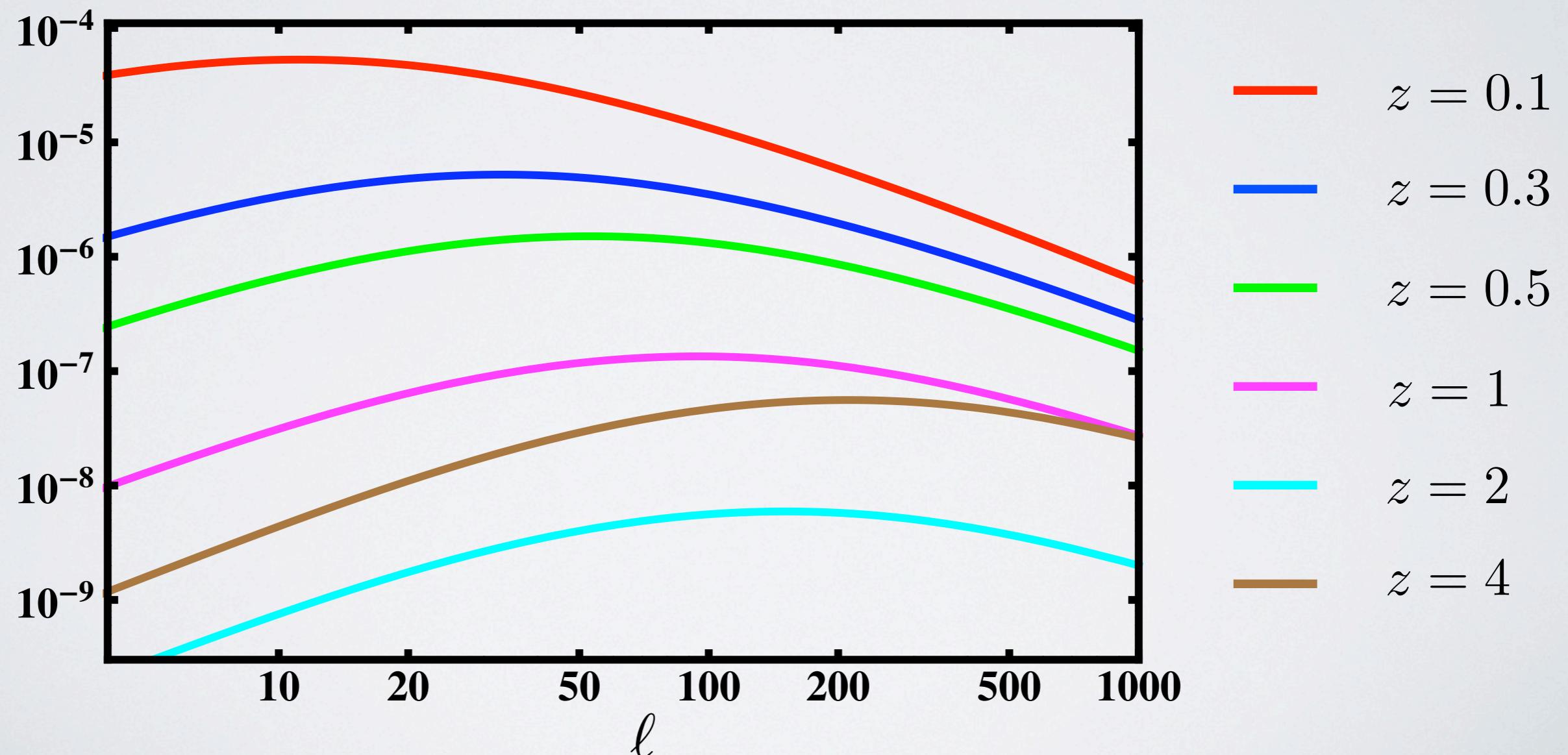


Peculiar velocities on the luminosity distance

$$\frac{\delta d_L}{d_L} = \left(1 - \frac{1}{\mathcal{H}_S \chi_S}\right) \mathbf{v}_S \cdot \mathbf{n}$$

$$\frac{\ell(\ell+1)}{2\pi} C_\ell^{(d_L)}$$

1% effect on the
luminosity distance

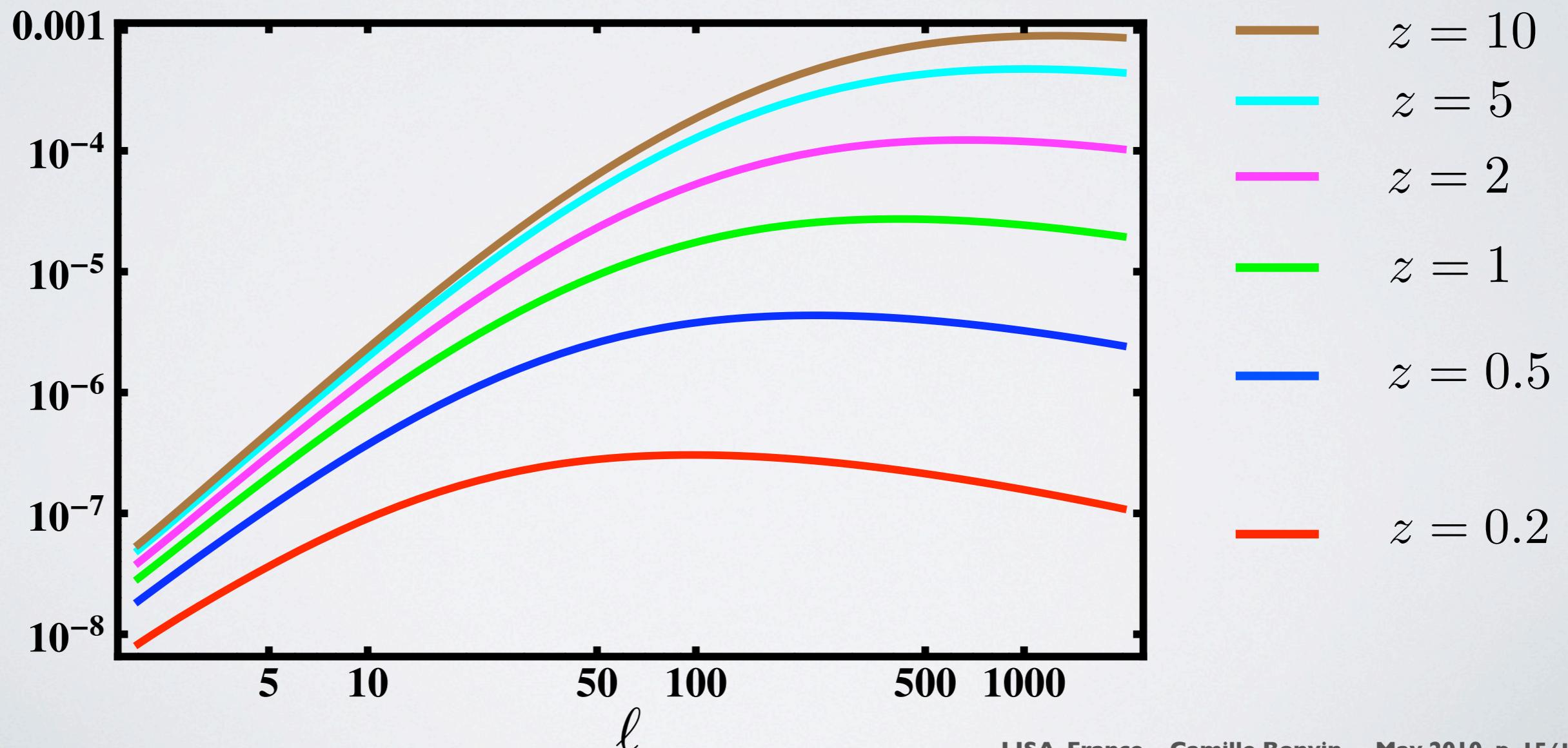


Lensing on the luminosity distance

$$\frac{\delta d_L}{d_L} = \int_0^{\chi_S} d\chi \frac{(\chi - \chi_S)\chi}{\chi_S} \Delta_{\perp} \Psi$$

$$\frac{\ell(\ell+1)}{2\pi} C_{\ell}^{(d_L)}$$

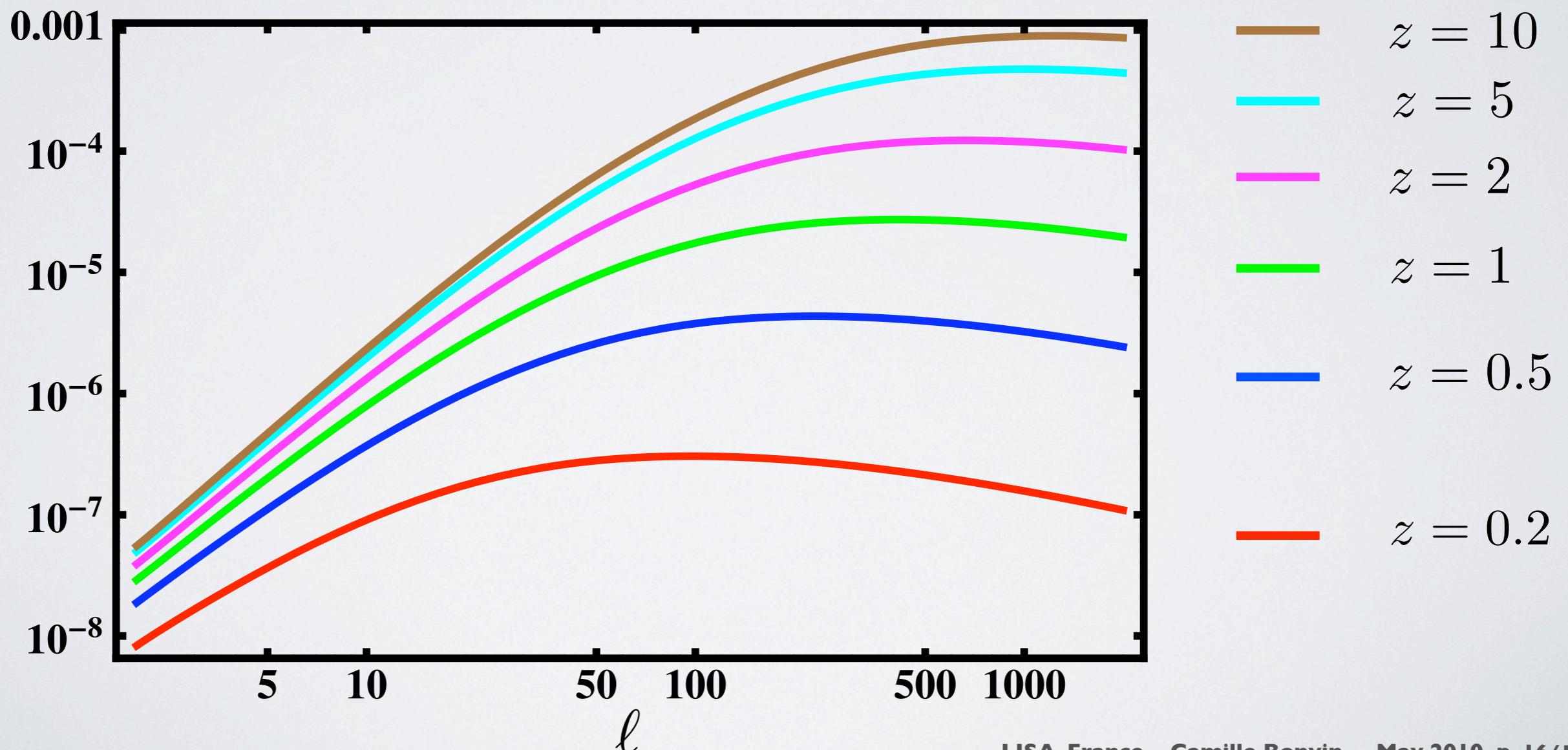
3% effect on the
luminosity distance



Lensing on the luminosity distance

With non-linearities: 5 – 10% effect on the luminosity distance. Holz and Hughes, 2005

It can be reduced by a factor 2-3, using the non-gaussian feature of the distribution. Hirata, Holz and Cutler, 2010

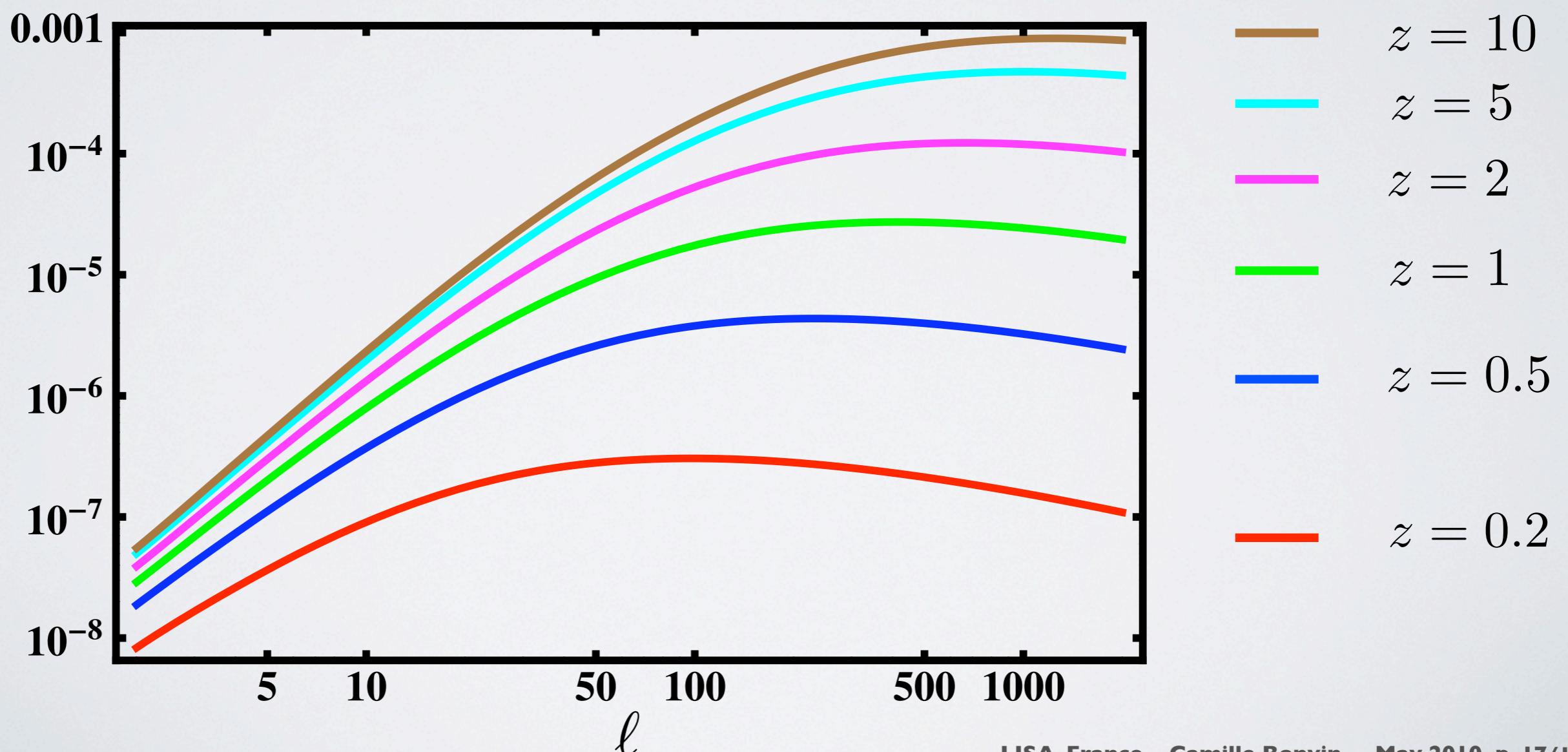


Lensing on the luminosity distance

With non-linearities: 5 – 10% effect on the luminosity distance. Holz and Hughes, 2005

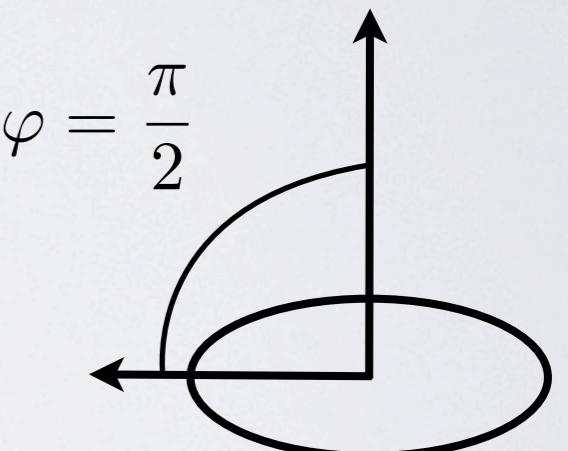
Modelling the magnification from galaxy distribution can reduce the error by a factor $\sqrt{3}$

Gunnarsson et al., 2005



Other effects

- ◆ The deflection has an impact on the **position** of the source in the sky $\nabla_{\perp} \Psi$
- ◆ The deflection has an impact on the **orientation** of the binary system with respect to us: even if we see the system edge-on, we will observe a small h_x component.



Conclusion

- ◆ Large-scale structure affect both the **redshift** and the **luminosity distance** → they affect the amplitude and the frequency of the wave, and the redshifted chirp mass.
- ◆ At small redshift, the dominant contribution is due to **peculiar velocities**. They have an impact on the redshift and on the luminosity distance.
- ◆ At large redshift, the dominant contribution is due to gravitational **lensing**, that affects only the luminosity distance.